Abstract— The electromagnetic wave propagation in the mobile communication environments can be predicted by using ray tracing approach which incorporates reflection and diffraction. However, the diffuse scattering effects are often non-negligible in the experiments. This is because when the energy is scattered to all the directions when electromagnetic wave impinges on a rough surface e.g. building wall. In this paper we present Method Of Moments (MOM) analysis of the scattering from the 2D rough surface model of the building. This result of scattering is compared to that for Physical Optics Method (PO).

Keywords— Wave Scattering, Building Surface, Method of Moment, Physical Optics

I. INTRODUCTION

The electromagnetic wave propagation can be predicted by using ray tracing approach which incorporates reflection and diffraction. This approach is getting more popular in the propagation prediction and the channel modeling in the mobile communication system [1] [2]. In the urban area, the buildings are the main scatterers which determine the propagation properties. The deterministic and periodic rough surface that models the surface of more realistic buildings by using Physical Optics (PO) approximation has been studied [3]. The result proved that the PDF of specular reflection from the surface of the building can be well modeled by the Rician distribution. However, if the incident angle is large, the numerical estimation of the scattering by using PO approximation causes more errors. Due to the fundamental limitation of PO [5], scattering from different shape may be identical, because only the shape of the illuminated part of the scatterer affects the induced current $J^{PO}$.

This paper presents the numerical estimation of the electric field scattering from building surface by using Method of Moments for avoiding the error of the PO approximation. The fluctuation of the field strength due to the change of the specular reflection point on the surface has been evaluated by the cumulative distribution and the autocorrelation. The effect on the incident angle is also studied.

II. THE BASIC THEORY

A. Method of Moment

Assuming a two-dimensional structure which is uniform in the $z$-direction and the propagation vector of electromagnetic wave incident on this structure is within $xy$-plane with the electric field vector parallel to the $z$-axis is referred as the transverse magnetic (TM) case. The incident and scattered electric fields both satisfy the following scalar Helmholtz equation.

$$\nabla^2 E_z + k^2 E_z = j\omega \mu J_z,$$

where $k$ is the wave number equal to $2\pi/\lambda$ with $\lambda$ the wavelength of the incident field and $J_z$ is the primary current for the incident field and the induced surface current for the scattered field. The total field and primary source will be Kirchoff-Huygen principle as follows.

$$E_z(r) = j\omega \mu \int S J_z(\rho')G(\rho, \rho')dS + j\omega \mu \oint_S (\nabla' E_z(\rho') - \nabla' G(\rho, \rho')E_z(\rho)) \cdot \hat{n} dl$$

where $\partial S$ is the surface of the scatterer and $S$ is the domain of source and can be reduced to $l$ for scattered wave. Using the boundary condition on $\partial S$ as,

$$E_z^{inc}(r) + E_z^{scat}(r) = 0, \quad r \in \partial S,$$

so that the electric field integral equation (EFIE) can be obtained,

$$E_z^{inc}(r) = -j\omega \mu \oint S J_z(\rho')G(\rho, \rho')dl$$

where,

$$J_z(\rho') = \nabla' E_z(\rho') \cdot \hat{n} = \frac{\partial E_z(\rho')}{\partial n'}$$

and the Green’s function for the 2D-scalar Helmholtz equation is

$$G(\rho, \rho') = -\frac{1}{4\pi} H_0^2(k|\rho - \rho'|).$$

The above integral equation (4) is solved numerically by applying the method of moments, thus the integral equation is discretized into a set of a finite number of unknowns. This discretization is realized by first expanding the unknown functions and then taking the inner product of the integral equation with a set of weighting functions [4].
In this paper, the point matching method is used with the pulse basis function as follows,

\[ J_z(\rho') = \sum_{i=1}^{N} a_i \delta(\rho' - \rho_i). \]  

(7)

After substituting eq. (7) into eq. (4) and weighted by the delta function \( \delta(\rho - \rho_i) \), \( l = 1, 2, \ldots, N \), the following set of the linear equation are obtained.

\[ \sum_{i=1}^{N} z_{li} a_i = b_l \quad l = 1, 2, 3, \ldots, N \]  

(8)

\[ z_{li} = \begin{cases} \frac{kZ_c}{4} [1 - \frac{2}{\pi} \ln(\frac{2k\Delta i}{\pi})] \Delta i, & i = l, \\ \frac{kZ_c}{4} \Delta H_0^{(2)}(k\rho_i), & i \neq l, \end{cases} \]  

(9)

where \( Z_c \) is characteristic impedance of the free space.

B. Physical Optics

In the Physical Optics approximations, the induced current on the scatterer is approximated by the incident field. Physical optics approximation is known to give the good approximation results for the lit region. Because the specular reflection is most important in the ray-tracing, the approximation results for the lit region. Because the specular reflection point at around 515 cm. These differences are characteristic impedance of the free space.

\[ J^{PO}_z(\rho') = \frac{2}{\pi} \frac{\rho}{\rho^2} \phi(k) \]  

where \( \rho \) is modified Bessel function of the first kind and \( \phi(k) \) is the peak amplitude of the constant signal, and \( \sigma^2 \) is the average power of Rayleigh component. To characterize the Rician distribution, the following Rician factor is used.

\[ K = \frac{s^2}{2\sigma^2} \]  

(15)

The fluctuation frequency of the signal strength is evaluated by autocorrelation function. This function is mathematically defined as

\[ \phi(k) = \frac{1}{N - k} \sum_{i=0}^{N-k} x(i)x(i+k), \]  

(16)

where \( x(i) \) represents the \( i \)-th signal strength, and \( k \) is the index of shifting for reflection point. In this simulation, the autocorrelation \( \phi(k) \) is normalized by \( \phi(0) \).

B. The Result of Simulation

Figure 2 compares the results of scattered field obtained by the Method of Moment and the Physical Optics approximation. The field strength is normalized by that for plane conductor. It is shown that there is significant difference reflection point at around 148 cm and repeated periodically reflection point at around 515 cm. These differences are characteristic impedance of PO as mentioned before in the introduction section. The original CDF together with the Rician fitting result are presented in Fig. 3. The average loss of \(-9.64 \text{ dB} \) and Rician factor of the \(-4.13 \text{ dB} \) are obtained. Figure 4 shows autocorrelation of the scattered field.
strength of MOM in Fig. 2. The fluctuation of the field strength is found to be quasi-periodic with the period of 92 cm.

![Reflection point (cm) vs. dB](image)

**Fig. 2.** Scattered field strength for incident angle of 45°.

![Cumulative distribution of signal strength](image)

**Fig. 3.** Cumulative distribution of signal strength for 45° incidence.

![Autocorrelation of the scattered field](image)

**Fig. 4.** Autocorrelation of the scattered field for 45° incidence.

Figure 5 shows the scattered field strength for the fluctuation due to incident angles of 20°, 45° and 60°. It is shown that field strength with incident angle of 60° is more fluctuated than with the incident angle of 20°. Rician parameters and quasi-period are compared in table 1. It is noted if the field strength are more fluctuated then the Rician factor becomes small.

### IV. Conclusion

This paper presented the numerical estimation of the scattered field fluctuation from the 2-dimensional model PEC rough surface of the building. The scattered field is estimated by Moment of Method for avoiding the error of the PO approximation. The Numerical result proves that difference between the scattered field from MOM and PO approximation is small. The PDF of specular reflection from the surface of the building can be well modeled by Rician distribution. The result of autocorrelation suggests the quasi-periodicity of the scattering. The field strength of 60° incident angle is more fluctuated than the 20° incident angle.

### V. Future Works

The Method of Moment is good for the small surface but it is time consuming for a larger surface. In this case, it suggested to use some other fast method. 3-dimensional model should be applied with realistic problem in the future, as well.

### References


