Performance Enhancement of Cyclostationarity Detector by Utilizing Multiple Cyclic Frequencies of OFDM signals

Minseok Kim, Po Kimtho, and Jun-ichi Takada,

***Abstract*— Spectrum sensing is a key technical challenge for the cognitive radio (CR) technology which allows it to access the spectrum of the licensed systems without causing interference to them. It is well known that cyclostationarity detectors have a great advantage of the robustness of noise uncertainty which significantly degrades the performance and makes its implementation difficult in energy detectors. This paper pays attention to the fact that cyclostationarity detector can achieve diversity gain by manipulating multiple cyclic autocorrelation functions (CAFs). While most of combining schemes in cooperative sensing with multiple detectors require the signal to noise power ratio estimation which is impractical in low SNR regime, combining with multiple CAFs in single detector based on transmit signal characteristic can be used. In this paper, three detector schemes of selection combining (SC), equal gain combining (EGC) and maximum ratio combining (MRC) with multiple CAFs are evaluated in additive white Gaussian noise (AWGN) channel considering the OFDM signal of Japanese digital television broadcasting (ISDB-T) as a primary system. The numerical results will show that the cyclic detector based on MRC using several cyclic frequencies has the best detection performances even though EGC shows slightly worse performance in assumed condition.**

I. Introduction

Recently, a cognitive radio (CR) technology, which is a solution to a problem of spectrum scarcity, has received an increasing attention [1], [2]. For example, the IEEE 802.22 working group (WG) in the United States (US) has developed a standard of a fixed wireless regional area network (WRAN) based on CR technology for operating in the TV bands [3], [4]. The IEEE WRAN system is considered as the secondary system which can access the primary bands without creating harmful interference to the primary systems, e.g., TV systems. The key challenge in designing WRAN based CR is to detect the primary signals as reliable as possible, even in very low signal to noise ratio (SNR) regime. Therefore, spectrum sensor or signal detector is seen as an essential functionality of the cognitive WRAN system. Various signal detectors have been investigated for that purposes such as matched filter, energy detector, and cyclostationarity detector (hereafter, cyclic detector) utilizing cyclostationarity, etc.

A matched filter is usually considered as an optimal detector if the primary signal is perfectly known. In addition, this filter has to precisely demodulate the primary signal by performing timing and symbol synchronization which is almost impractical in very low SNR regime. An energy detector is simple to implement and does not need any prior knowledge about the primary signal but it is difficult to control false alarm rate because the statistics of the signals, noise and interference are not distinct in the signal processing [5]. A cyclic detector utilizing cyclostationarity is seen as a possible candidate to achieve the sensing requirements in the cognitive radio system because it does not need a explicit knowledge of noise distribution thus it is robust to random noise in practice. However, this technique supposes knowledge of a minimum characteristic of the primary signal such as modulation type, symbol rates and so on. Although the cyclic detector has a rich literature review, the result may have not been directly applicable to the cognitive radio system [6–9]. Recently a number of studies have extensively applied the cyclostationarity to signal detection in the cognitive radio application [10–18].

The signal detector can be considered as a binary hypothesis testing problem; the primary signal is absent ($\mathcal{H}_0$) or the primary signal is present ($\mathcal{H}_1$). The test statistic under $\mathcal{H}_0$ and $\mathcal{H}_1$ can be established by general likelihood ratio test (GLRT) which introduces the mean square sense consistency and the asymptotically complex normality of cyclic autocorrelation function (CAF) of the received signal [9]. The periodic nature of the cyclostationarity is exhibited by multiple cyclic frequencies and the CAFs at different cyclic frequencies are asymptotically uncorrelated [13], [19]. Although multiple CAFs at single time delay were utilized in [9], $\mathcal{H}_0$ or $\mathcal{H}_1$ can be determined by mutiple CAFs both at multiple cyclic frequencies and multiple time delay [12], [13]. In other words, cyclic detector can achieve diversity gain by manipulating multiple CAFs. In [10], [11], the detection performances were evaluated at any arbitrary cyclic frequencies which might result poor detection performances. And only selection and equal gain combining were discussed in [12], [13] where comprehensive guideline about how to select cyclic frequencies for evaluating the detection performances was not provided.

While most of combining schemes in cooperative sensing with multiple detectors require the signal to noise power ratio (SNR) estimation which is impractical in low SNR regime, a single detector combining test statistics obtained at multiple cyclic frequencies based on transmit signal characteristic can be used. In this paper, the detection performances are evaluated in three combining schemes of selection combining (SC), equal gain combining (EGC) and maximum ratio combining (MRC). The probability of detection (P_{D}) vs. SNR and P_{D} vs. probability of false alarm (P_{FA}) are examined. The performance is evaluated based on numerical simulations observed in an additive white Gaussian noise (AWGN) by considering an orthogonal frequency division multiplexing (OFDM) signal of Japanese digital television broadcasting (ISDB-T) as the primary users. In addition, we also compare the performance of cyclic detector with well known energy detector.

The rest of this paper is organized as follows. Preliminaries about cyclostationarity detector including cyclostationary property of OFDM signal and test statistic and detection rule for cyclic detector are presented in Sect. 2. In Sect. 3, a description of signal detection methods based on above three schemes are presented. The simulation results are illustrated in Sect. 4. Finally, Sect. 5 concludes this paper.

II. Preliminaries

A. Cyclic Autocorrelation Function (CAF)

Consider a zero mean discrete time signal $x[n] = x(nT_{\rm o})$, where $T_{\rm o}$ is the sample period. A signal $x[n]$ exhibits a wide sense second order cyclostationarity if its time varying autocorrelation function

$$R_{xx}[n,l] = E\{x[n]x^{\star}[n+l]\}, \tag{1}$$

is periodic in terms of a fixed lag $l(=0,\pm1,\pm2,\cdots)$. $x[n]$ for $n = 0,\cdots,N-1$ represents a sample of the signal $x[n]$ and N denotes the number of samples. We assume that $T_{\rm s}$ is a period of $R_{xx}[n,l]$. $T_{\rm s}$ corresponds to embedded periodicity in the signal $x[n]$ e.g., symbol rate, carrier frequency. From the Fourier series expansion, that is

$$R_{xx}[n,l] = \sum_{k} R_{xx}^{\alpha_k}[l] e^{j2\pi\frac{nk}{N}}, \tag{2}$$

where α_k is called cyclic frequency of $x[n]$ for $k = 0,\pm1,\pm2,\cdots$ (k is called cyclic frequency index). Typically, α_k is related to symbol rate, modulation scheme and carrier frequency of $x[n]$. The Fourier coefficient $R_{xx}^{\alpha_k}[l]$ is called a cyclic autocorrelation function (CAF) at cyclic frequency α_k and at time lag l and written by

$$R_{xx}^{\alpha_k}[l] = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} R_{xx}[n,l] e^{-j2\pi\frac{nk}{N}}, \tag{3}$$

and in practice it can be estimated as

$$\hat{R}_{xx}^{\alpha_k}[l] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x^{\star}[n+l] e^{-j2\pi\frac{nk}{N}} \tag{4}$$

[9]. $\hat{R}_{xx}^{\alpha_k}[l]$ is an estimate of conjugate CAF of $R_{xx}^{\alpha_k}[l]$ and $\star$ denotes conjugate operator. If α_k is the cyclic frequency of $x[n]$, then $\hat{R}_{xx}^{\alpha_k}[l] \neq 0$. However, $\hat{R}_{xx}^{\alpha_k}[l] \neq 0$ although α_k is not a cyclic frequency of $x[n]$ because $\hat{R}_{xx}^{\alpha_k}[l]$ is computed using a finite number of samples, N. Therefore, it is quite difficult to conclude that α_k is the cyclic frequency of $x[n]$ by just checking the value of $\hat{R}_{xx}^{\alpha_k}[l]$ at cyclic frequency α_k and at time lag l. Therefore, it is necessary to determine the statistical test for the presence and the absence of the cyclostationarity.

B. Cyclic Autocorrelation Function of OFDM signal

Orthogonal Frequency Division Multiplexing (OFDM) is a key technology in the digital radio broadband transmission including the TV broadcasting systems. For examples, both Digital Video Broadcasting Terrestrial (DVB-T) system and Integrated Services Digital Broadcasting-Terrestrial (ISDB-T) system use OFDM [20]. Therefore, it is reasonable to assume that the primary signal is an OFDM signal. The problem of detecting an OFDM signal is thus very relevant. In this paper, the OFDM signal of ISDB-T system is considered as the primary signals. The complex baseband OFDM signal $s[n]$ can be represented as follows [20].

$$\begin{aligned} s[n] &= \sum_{u=-\infty}^{+\infty} g[n-uN_{\rm s}] \cdot \\ &\quad \left(\sum_{i=0}^{N_{\rm c}-1} d[u,i]\cdot e^{j2\pi\frac{i-(N_{\rm c}-1)/2}{N_{\rm u}}n}\right), \end{aligned} \tag{5}$$

where $N_{\rm c}$ and $N_{\rm u}$ denote the number of subcarrier and the number of FFT points, respectively. $N_{\rm s}$ $(= N_{\rm u}+N_{\rm g})$ becomes the total number of samples in an OFDM symbol by adding $N_{\rm g}$ samples of cyclic prefix. $d[n,i]$ is the transmit information for i-th subcarrier in n-th symbol, where they are statistically uncorrelated with each other and modulated by a quadrature amplitude modulation (64 QAM). $g[n]$ is the rectangular shaped pulse

$$g[n] = \begin{cases} 1, & \text{for } 0 \le n \le N_{\rm s} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

The autocorrelation function of $s[n]$ can be written by

$$\begin{aligned} R_{ss}[n,l] &= \sigma_d^2 A[l] \sum_{n=-\infty}^{\infty} g[n-uN_{\rm s}] \cdot \\ &\quad g[n-uN_{\rm s}+l], \end{aligned} \tag{7}$$

where $\sigma_d^2 = E\{d[n,i]d^{\star}[n,i]\}$ and $A[l]$ can be expressed as

$$\begin{aligned} A[l] &= \sum_{i=0}^{N_{\rm c}-1} e^{-j2\pi[(i-(N_{\rm c}-1)/2)/N_{\rm u}]l} \\ &= \frac{\sin(\pi l N_{\rm c}/N_{\rm u})}{\sin(\pi l/N_{\rm u})}. \end{aligned} \tag{8}$$

From (7) it is clear that the autocorrelation function is periodic with period of $N_{\rm s}$ at lag l. Thus the OFDM signal is cyclostationary with cyclic frequency

$$\mathcal{A} = \{\alpha_k \mid \alpha_k = \frac{k}{T_{\rm s}} = \frac{k}{N_{\rm s}T_{\rm o}}, k = 0,\pm1,\pm2,\cdots\}. \tag{9}$$

The CAF of $g[n]$, $R_{gg}^{\alpha}[l]$, can be calculated by Fourier series expansion of (7) by

$$R_{gg}^{\alpha_k}[l] = \begin{cases} \frac{\sin[\pi\alpha_k(N_{\rm s}-|l|)]}{\pi k}, & \text{for } |\, l \,| \le N_{\rm s} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Hence, $R_{ss}^{\alpha_k}[l]$ can therefore be written as

$$R_{ss}^{\alpha_k}[l] = \begin{cases} \sigma_d^2 A[l] \frac{\sin[\pi\alpha_k(N_{\rm s}-|l|)]}{\pi k}, & \text{for } |\, l \,| \le N_{\rm s} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

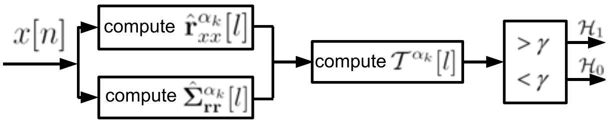

Fig. 1. Cyclic detector using cyclostationary property.

(11) establishes two important facts. First, $R_{ss}^{\alpha_k}[l] \neq 0$ since $A[l] \neq 0$ when $l = \pm N_\mathrm{u}$ and it is noted that $R_{ss}^{\alpha_k}[l]$ has the largest CAFs among those at other lags. Second, the OFDM signal must insert the guard interval at the start of the useful OFDM symbol otherwise $R_{ss}^{\alpha_k}[l] = 0$ because when $N_\mathrm{s} = N_\mathrm{u}$ and $l = N_\mathrm{u}$ leading to $\sin[\pi\alpha_k(N_\mathrm{s} - \mid l \mid)] = 0$. Therefore, the OFDM signal induces cyclostationarity when the cyclic prefix is used in the OFDM waveform and $R_{ss}^{\alpha_k}[l]$ has the largest CAFs when l is equal to N_u.

C. Statistical Test of Cyclic Detector

In this paper, we basically follow the same approach as [9]. In OFDM signals, cyclic frequencies appear only at the time lags of integer multiples of useful symbol duration ($\tau = T_u$, namely $l = N_\mathrm{u}$), thus we consider CAFs in single time lag at $l = N_\mathrm{u}$ where the CAFs are dominant, although Dandawate's approach generally used CAFs in multiple time lags. By using the mean square sense consistency and asymptotical complex normality of CAF of the received signal, the statistical test based on GLRT for the presence and the absence of the cyclostationarity has been established as follows [9]

$$\mathcal{T}^{\alpha_k}[l] = N \cdot \hat{\mathbf{r}}_{xx}^{\alpha_k}[l] \cdot \hat{\Sigma}_{\mathbf{rr}}^{\alpha_k}[l]^{-1} \cdot \hat{\mathbf{r}}_{xx}^{\alpha_k}[l]^T, \tag{12}$$

where $\hat{\mathbf{r}}_{xx}^{\alpha_k}[l]$ is a row vector for single CAF which is defined by

$$\hat{\mathbf{r}}_{xx}^{\alpha_k}[l] = [\mathrm{Re}\{\hat{R}_{xx}^{\alpha_k}[l]\}, \mathrm{Im}\{\hat{R}_{xx}^{\alpha_k}[l]\}], \tag{13}$$

and $\hat{\Sigma}_{\mathbf{rr}}^{\alpha_k}[l]$ is the estimation of the covariance matrix which can be derived analytically as follows [9]

$$\hat{\Sigma}_{\mathbf{rr}}^{\alpha_k}[l] = \begin{bmatrix} \mathrm{Re}\left\{\frac{\hat{S}_f^{\alpha_k}[l] + \hat{S}_f^{\alpha_k\star}[l]}{2}\right\} & \mathrm{Im}\left\{\frac{\hat{S}_f^{\alpha_k}[l] - \hat{S}_f^{\alpha_k\star}[l]}{2}\right\} \\ \mathrm{Im}\left\{\frac{\hat{S}_f^{\alpha_k}[l] + \hat{S}_f^{\alpha_k\star}[l]}{2}\right\} & \mathrm{Re}\left\{\frac{\hat{S}_f^{\alpha_k\star}[l] - \hat{S}_f^{\alpha_k}[l]}{2}\right\} \end{bmatrix}, \tag{14}$$

where the cyclic spectra

$$\hat{S}_f^{\alpha_k}[l] = \frac{1}{NL} \sum_{s=-(L-1)/2}^{(L-1)/2} W_L[s] F_{N,l}[k-s] \cdot F_{N,l}[k+s], \tag{15}$$

and

$$\hat{S}_f^{\alpha_k\star}[l] = \frac{1}{NL} \sum_{s=-(L-1)/2}^{(L-1)/2} W_L[s] F_{N,l}^{\star}[k+s] \cdot F_{N,l}[k+s], \tag{16}$$

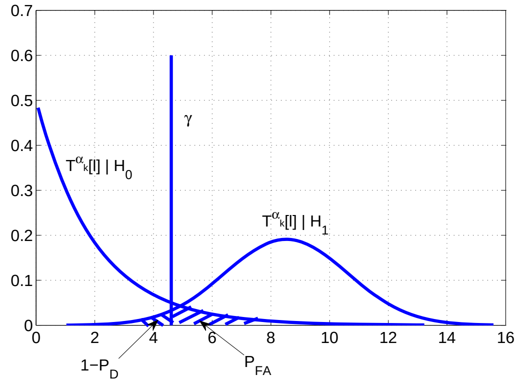

Fig. 2. Distribution of $\mathcal{T}^{\alpha_k}[l]$ under $\mathcal{H}_0$ and $\mathcal{H}_1$ along with the threshold γ

where $W_L[s]$ is a smoothing spectral window (e.g. Kaiser window) and $F_{N,l}[k]$ is the Fourier transform of $f[n] = x[n]x^\star[n+l]$ which is defined as,

$$F_{N,l}(k) = \sum_{n=0}^{N-1} x[n]x^\star[n+l]e^{-j2\pi\frac{nk}{N}}. \tag{17}$$

D. Detection Rule of Cyclostationarity Detector

The signal detection problem can be modeled as two binary hypotheses test as follows,

$$\begin{cases} \mathcal{H}_0 : x[n] = w[n] & : \text{signal absence} \\ \mathcal{H}_1 : x[n] = s[n] + w[n] & : \text{signal presence} \end{cases}, \tag{18}$$

where $x[n]$ represents the sample of the received signal, $s[n]$ is the primary signal which may be modeled as a zero mean cyclostationary signal and $w[n]$ denotes the complex additive white Gaussian noise (AWGN) process. The detection problem aims at determining which of $\mathcal{H}_0$ or $\mathcal{H}_1$ is the most likely.

Figure 1 depicts the detection algorithm using cyclostationarity with the test statistic $\mathcal{T}^{\alpha_k}[l]$ in (12). In order to design the cyclic detector, at first, the distribution of $\mathcal{T}^{\alpha_k}[l]$ under $\mathcal{H}_0$ should be known so that the threshold value γ can be determined for a specified probability of false alarm (P_FA). The signal detection mechanism can be stated as follows. The signal presence and absence are determined by comparing the test statistic of $\mathcal{T}^{\alpha_k}[l]$ against a predefined threshold γ. When $\mathcal{T}^{\alpha_k}[l] \geq \gamma$, the primary signal is decided to be present and when $\mathcal{T}^{\alpha_k}[l] < \gamma$ the primary signal is decided to be absent.

Figure 2 illustrates the distribution of $\mathcal{T}^{\alpha_k}[l]$ under $\mathcal{H}_0$ and $\mathcal{H}_1$. As described in Sect.II, under $\mathcal{H}_0$ it is assumed that each elements of $\hat{\mathbf{r}}_{xx}^{\alpha_k}[l]$ is asymptotical Gaussian random variable with zero mean, thus $\mathcal{T}^{\alpha_k}[l]$ follows chi-square distribution with degree of two, namely

$$\lim_{N\to\infty} \mathcal{T}^{\alpha_k}[l] \overset{D}{=} \chi_2^2, \tag{19}$$

regardless of the noise variance. On the other hand, under $\mathcal{H}_1$, it has been shown that $\mathcal{T}^{\alpha_k}[l]$ asymptotically follows non-central chi-square distribution with degree of two, namely

$$\lim_{N\to\infty} \mathcal{T}^{\alpha_k}[l] \overset{D}{=} \chi_2^2(N \cdot \mathbf{r}_{xx}^{\alpha_k}[l] \cdot \Sigma_{\mathbf{rr}}^{\alpha_k}[l]^{-1} \cdot \mathbf{r}_{xx}^{\alpha_k}[l]^T) \tag{20}$$

[9], where $\mathbf{r}_{xx}^{\alpha_k}[l]$ and $\Sigma_{xx}^{\alpha_k}[l]$ denote true value of CAF and covariance matrix. As can be seen, there is always a tradeoff between having a high probability of detection ($P_{\rm D}$) and having a low probability of false alarm $P_{\rm FA}$. This tradeoff can be made by changing the detection threshold. $P_{\rm FA}$ can be derived as follows

$$P_{\rm FA} = \Pr\{\mathcal{T}^{\alpha_k}[l] > \gamma \mid \mathcal{H}_0\}. \tag{21}$$

The decision threshold γ can be defined as

$$\gamma = F_2^{-1}\left(1 - P_{\rm FA}\right), \tag{22}$$

where $F_2^{-1}(\gamma)$ is the inverse function of $F_2(\gamma) = \int_0^\gamma \frac{1}{2}\exp(-\frac{y}{2})dy$, the cumulative distribution function of chi-square distribution with degree of two. Once the threshold has been set, one can theoretically evaluate the probability of detection such that

$$P_{\rm D} = \Pr\{\mathcal{T}^{\alpha_k}[l] \geq \gamma \mid \mathcal{H}_1\}. \tag{23}$$

However, in reality, the Dandawate's approach assumed to use the asymptotical property of the test statistic where there was no consideration of the SNR, but the SNR significantly affects on the distribution in practice. Further this asymptoticity makes the theoretical analysis of the performance in terms of SNR quite difficult because the test statistic can use only limited number of samples.

III. Combining Methods with Multiple Cyclic Spectrum Components

As mentioned above, the periodic nature of the cyclostationarity in OFDM signal is exhibited by multiple cyclic frequencies. Thus, $\mathcal{H}_0$ or $\mathcal{H}_1$ can be determined by using multiple CAFs at multiple cyclic frequencies which are asymptotically uncorrelated with each other. In other words, cyclic detector can achieve diversity gain by manipulating multiple CAFs, e.g., at $\alpha_1 = \frac{1}{T_{\rm s}}$, $\alpha_2 = \frac{2}{T_{\rm s}}$, $\cdots$,$\alpha_{N_\alpha} = \frac{N_\alpha}{T_{\rm s}}$. In this scheme, a secondary user combines multiple test statistics of $\mathcal{T}^{\alpha_k}[l]$ over different cyclic frequencies, which are also asymptotically uncorrelated non-central chi-square random variables [13], [19]. In other word, a weighted sum extension of test statistics can be used as

$$\mathcal{T}^{\bar{\mathcal{A}}} = \sum_{k=-N_\alpha}^{N_\alpha} w_k \mathcal{T}^{\alpha_k} = \boldsymbol{T}^T \boldsymbol{w}, \tag{24}$$

where

$$\boldsymbol{T} = [\mathcal{T}^{\alpha_1}, \mathcal{T}^{\alpha_2}, \cdots, \mathcal{T}^{\alpha_{N_c}}]^T, \tag{25}$$

$$\boldsymbol{w} = [w_1, w_2, \cdots, w_{N_\alpha}]^T, \tag{26}$$

where w_k is real-valued weight coefficient. $\bar{\mathcal{A}}$ ($\subset \mathcal{A}$) denotes

$$\bar{\mathcal{A}} = \{\alpha_k \mid k = 0, \pm 1, \pm 2, \cdots, \pm N_\alpha\}. \tag{27}$$

A. Selection Combining (SC)

The key idea behind the SC technique is that the detector monitors the value of $\mathcal{T}^{\alpha_k}[l]$ at all $2N_\alpha + 1$ spectral lines at a time and select the spectral lines with the highest decision statistic. Thus, the test statistic for SC scheme can be constructed as

$$\mathcal{T}_{\rm SC}^{\bar{\mathcal{A}}}[l] = \max_{\bar{\mathcal{A}}} \mathcal{T}^{\alpha_k}[l] \tag{28}$$

The implementation of SC technique can be found in [21]. Under $\mathcal{H}_0$, using the cumulative distribution function (CDF) of $\mathcal{T}_{\rm SC}^{\bar{\mathcal{A}}}[l]$ for given independent $\{\mathcal{T}^{\alpha_k}[l]\}_{k=-N_\alpha}^{N_\alpha}$ variables, the $P_{\rm FA}$ can be written as

$$P_{\rm FA} = 1 - F_2\left(\gamma_{\rm SC}\right)^{2N_\alpha+1}, \tag{29}$$

where $\gamma_{\rm SC}$ is the decision threshold for SC which can be written as

$$\gamma_{\rm SC} = F_2^{-1}\left(\left(1 - P_{\rm FA}\right)^{\frac{1}{2N_\alpha+1}}\right). \tag{30}$$

B. Equal Gain Combining (EGC)

Instead of choosing maximum test statistic, combination of test statistics with equal gain is considered. Under $\mathcal{H}_0$, the test statistic at each CAF are assumed to be independent. In the test statistic of EGC technique is calculated with $w_k = 1/\sqrt{2N_\alpha + 1}$ for all k. Under $\mathcal{H}_0$ the test statistic of EGC can be approximately written as

$$\mathcal{T}_{\rm EGC}^{\bar{\mathcal{A}}}[l] = \frac{1}{\sqrt{2N_\alpha + 1}} X, \tag{31}$$

where the random variable $X \sim \mathcal{X}^2_{2(2N_\alpha+1)}$. The threshold $\gamma_{\rm EGC}$ can be given as $P_{\rm FA} = \Pr\{\mathcal{T}_{\rm EGC}^{\bar{\mathcal{A}}}[l] > \gamma_{\rm EGC} \mid \mathcal{H}_0\}$. Similar to (22), the $\gamma_{\rm EGC}$ can be calculated as

$$\gamma_{\rm EGC} = F^{-1}_{2(2N_\alpha+1)}(1 - P_{\rm FA}), \tag{32}$$

where $F^{-1}_{2(2_{N_\alpha}+1)}$ is the inverse function of $F_{2(2_{N_\alpha}+1)}$, the cumulative distribution function of chi-square distribution with degree of $2(2_{N_\alpha} + 1)$.

C. Maximum Ratio Combining (MRC)

In optimizing the $P_{\rm D}$ performance with multiple test statistics, there can be some techniques, e.g., the optimization of combined PDF (probability density function) both for $\mathcal{H}_0$ and $\mathcal{H}_1$ simultaneously. To measure the effect of the PDF on the detection performance, a modified deflection coefficient was introduced in [23]. In cyclic detector $\mathcal{T}^{\alpha_k}$ follows approximately chi-square distribution with degree of two under $\mathcal{H}_0$, and non-central chi-square distribution of $\chi_2^2(\mathcal{T}_0^{\alpha_k})$ under $\mathcal{H}_1$ where $\mathcal{T}_0^{\alpha_k}$ denotes a asymptotic test statistic value. However the distribution under $\mathcal{H}_1$ is quite difficult to formulate because the non-centrality strongly depends on the SNR as well as the computational gain. It should be noted that Dandawate's approach assumed the asymptotical property of the distribution, thus there was no consideration of any noise contribution. In this paper, we propose an alternative method of maximum ratio combining to enhance the performance, that is, spectrum sensing scheme using CAF values of transmit signal.

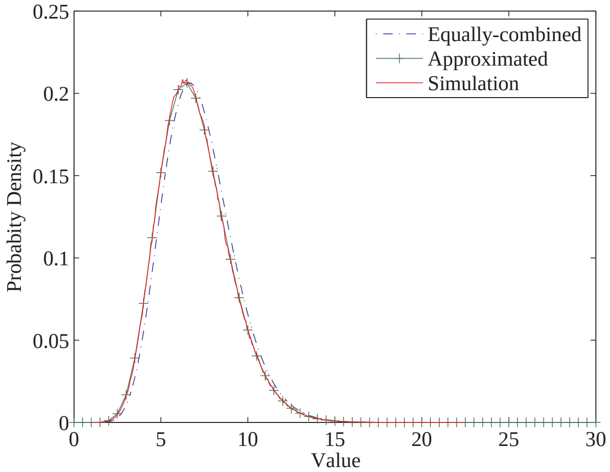

Fig. 3. Approximation of weighted and sumed PDF of 13 chi-square distributions under $\mathcal{H}_0$ where $\boldsymbol{w}^T\boldsymbol{w} = 1$.

In multiple antenna system, the maximum ratio combining technique is a commonly used technique to combine the output signal maximizing the SNR with multiple received signals. As applied in [22], MRC scheme has been proposed for cooperative sensing among multiple sensors. However most of combining schemes in cooperative sensing with multiple detectors require the signal to noise power ratio (SNR) estimation which is impractical in low SNR regime and fading environment. On the other hand, a single detector combining test statistics obtained at multiple cyclic frequencies can be based on transmit signal characteristic because the fluctuation of individual cyclic frequency spectrum taken at an instant bears no straightforward relation to the propagation channel effect.

A simple strategy in MRC detector is to combine the output with multiple test statistics at different cyclic frequencies according to the non-centrality value of the distribution in (20), that is, magnitude ratio of asymptotic test statistic of $\mathcal{T}_0^{\alpha_k}[l] = N\mathbf{r}_{xx}^{\alpha_k}[l]\Sigma_{\mathbf{rr}}^{\alpha_k}[l]^{-1}\mathbf{r}_{xx}^{\alpha_k}[l]^T$ which is a quadratic value proportional to $|R_{ss}^{\alpha_k}|^2$. Namely, we can obtain the MRC weight as

$$w_k = \frac{\mathcal{T}_0^{\alpha_k}[l]}{\sqrt{\sum_{-N_\alpha}^{N_\alpha}\left(\mathcal{T}_0^{\alpha_k}[l]\right)^2}}, \tag{33}$$

where $\boldsymbol{w}^T\boldsymbol{w} = 1$. Practically the MRC weight can be obtained by using the theoretical calculation of CAFs with known primary signal because (33) requires only the ratio among different CAFs. The CAFs for an OFDM signal was described in Sect. II and $|R_{ss}^{\alpha_k}|^2$ of (11) can be actually used.

Under $\mathcal{H}_0$, the test statistic combined by MRC is

$$\mathcal{T}_{\mathrm{MRC}}^{\bar{\mathcal{A}}}[l] \;=\; \sum_{k=-N_\alpha}^{N_\alpha} w_k \mathcal{T}^{\alpha_k}[l], \tag{34}$$

where $\mathcal{T}^{\alpha_k} \sim \chi_2^2$. (34) becomes weighted sum of the central chi-square distribution. In order to calculate the threshold value the closed form expression of PDF for (34) is needed. Instead of the exact expression, (34) can be alternatively approximated by

$$\mathcal{T}_{\mathrm{MRC}}^{\bar{\mathcal{A}}}[l] \sim a \cdot \chi_b^2, \tag{35}$$

as applied in [24], where a and b are the scale factor and the approximated degree of freedom. By using the first and second moments of (34) and (35) as

$$\mathrm{E}[\mathcal{T}_{\mathrm{MRC}}^{\bar{\mathcal{A}}}[l]|\mathcal{H}_0] \;=\; \sum_{k=-N_\alpha}^{N_\alpha} w_k \cdot 2 = a \cdot b, \tag{36}$$

$$\mathrm{Var}[\mathcal{T}_{\mathrm{MRC}}^{\bar{\mathcal{A}}}[l]|\mathcal{H}_0] \;=\; \sum_{k=-N_\alpha}^{N_\alpha} w_k^2 \cdot 4 = a^2 \cdot 2b, \tag{37}$$

we can obtained a and b as

$$a \;=\; \sum_{k=-N_\alpha}^{N_\alpha} w_k^2 / \sum_{k=-N_\alpha}^{N_\alpha} w_k, \tag{38}$$

$$b \;=\; 2\left(\sum_{k=-N_\alpha}^{N_\alpha} w_k^2\right)^2 / \sum_{k=-N_\alpha}^{N_\alpha} w_k. \tag{39}$$

Figure 3 shows an example of the approximated distribution that is combined by 13 central chi-square distributions with degree of two under $\mathcal{H}_0$ where $\boldsymbol{w}^T\boldsymbol{w} = 1$. As can be seen, this approximation provides sufficient accuracy to determine threshold value corresponding to the false alarm probability than that of equally-combined distribution.

Similarly to (22), the γ_{MRC} can be calculated as

$$\gamma_{\mathrm{MRC}} = a \cdot F_b^{-1}(1 - P_{\mathrm{FA}}), \tag{40}$$

where F_b^{-1} is the inverse CDF of F_b, chi-square distribution with degree of b.

IV. Simulation Results

A. Parameters

The detail of ISDB-T mode 3 OFDM signal specification is provided in [20]. Single ISDB-T channel basically consists of 13 OFDM segments for wide band and 1 or 3 OFDM segments for narrow band system. Now, this system serves in the frequency bands from 470 MHz to 770 MHz with the bandwidth of 5.572 MHz in Japan. In this paper, the ISDB-T Mode-3 signal which includes all 13 OFDM segments is considered. The OFDM parameters are presented in Table I. In addition, Table II shows the simulation parameters used for estimating the $\hat{R}_{xx}^{\alpha_k}[l]$ and $\hat{\Sigma}_{\mathbf{rr}}^{\alpha_k}[l]$ in constructing the test statistic $\mathcal{T}^{\alpha_k}[l]$.

B. CAF Estimation

Now, the CAF estimation in (4) is compared with the ideal CAF of the OFDM signal using (11). The $\hat{R}_{xx}^{\alpha_k}[l]$ in (4) is computed via FFT with size N and a fixed time lag $l = N_u$ as shown in Table II. The normalized spectrum of the ideal CAF and its estimation results are shown in Fig. 4 (a) and (b), respectively, where they are the square values of the magnitude and it is shown that their peaks appear at $\alpha_k = k/T_{\mathrm{s}}$, $k =$

TABLE I

OFDM SIGNAL PARAMETERS OF ISDB-T MODE-3

Parameters	Values
Modulation	64 QAM
OFDM useful symbol duration (T_{u})	1008 μs
OFDM guard interval (T_{g})	126 μs ($= T_{\mathrm{u}}/8$)
OFDM total symbol duration (T_{s})	1134 μs ($= T_{\mathrm{u}}$ + T_{g})
Number of sub-carriers (N_{c})	5617
Carrier separation (Δf)	0.9920 kHz ($= 1/T_{\mathrm{u}}$)
Sampling frequency (f_{s})	8.127 MHz
FFT size (N_{FFT})	8192

TABLE II

SIMULATION PARAMETERS FOR ESTIMATING TEST STATISTIC

Parameters	Values
Data length (N)	10 symbols (= $10N_{\mathrm{s}}$)
Time lag (l)	8192 (=N_u)
Kaiser window parameter	$L = 65537$, $\beta = 1$

$0, \pm1, \pm2, \cdots$. Thus OFDM signal exhibits cyclostationarity at cyclic frequencies $\alpha_k = k/T_{\mathrm{s}}$ as expected. To estimate the $\mathcal{T}^{\alpha_k}[l]$, first, we compute the row vector in (13). Second, we compute the $\hat{\Sigma}^{\alpha_k}_{\mathbf{rr}}[l]$ estimation in (14) by using (15) and (16). Here the non-conjugate and conjugate cyclic spectra of $f[n]$ in (15) and (16) can be computed via FFT with size N using Kaiser window with parameters shown in Table II. Finally, the test statistic $\mathcal{T}^{\alpha_k}[l]$ can be obtained by substituting (13) and (14) into (12). Fig. 4(c) shows the test statistic (12) without noise and it is seen that the $\mathcal{T}^{\alpha_k}[l]$ represents Fig. 4(a) well as expected. Further, as can be seen, some cyclic frequencies cannot be utilized because they might be quite lower than the decision threshold. For example, when $| \; k \; |> 7$ all the spectral lines are disappeared. Therefore, cyclic detector should detect the OFDM signal at α_k with $| \; k \; |< 7$ in order to gain a better detection performance.

C. Detection Performances

In our simulations, the noise level is fixed from trial to trial and the SNR is defined as $\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_s^2}{\sigma_w^2}\right)$ where σ_s^2 and σ_w^2 are the variances of the signal and noise, respectively. Detection was carried out using a signal recorded for a duration of 10 OFDM symbols (11.34 ms). The P_{FA} is specified at 10% as recommended in IEEE 802.22 working group and the simulation was carried out over $1,000$ realizations. Figure 5 shows the detection probabilities P_{D} vs. SNR for a single cyclic frequency. The performance is examined for several choices of α_k for illustrating the impact of this parameter on the detector. The result shows that the detection performance highly depends on the choice of cyclic frequencies. As expected, the detection probability at α_0 and α_1 is the best comparing with those at the other cyclic frequencies. The P_{D} approaches to 1 when the SNR gets close to -10 dB with the sensing time about 11.34 ms.

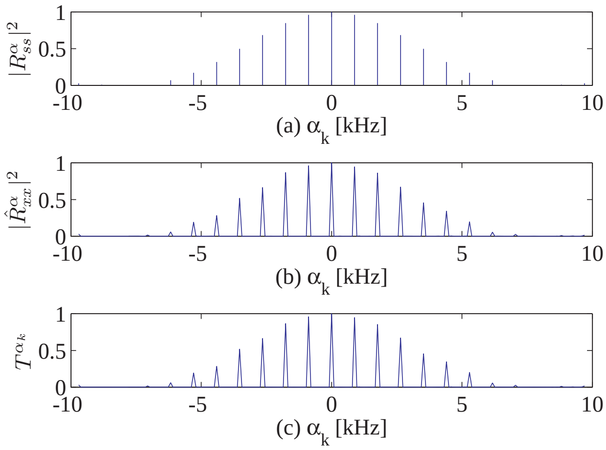

Fig. 4. Normalized power spectrum of (a) Ideal CAF, (b) CAF estimation, (c) test statistic where $l = N_u$.

Figure 6 illustrates the performance comparison between the cyclic detectors with SD (single detector), SC, EGC and MRC where the performance of the energy detector is also plotted as a reference. In this simulation, the multiple cyclic detector combined the test statistic with 13 CAFs of $-\alpha_6 \sim \alpha_6$. First, we can observe that MRC has the best performance which is slightly better than that of EGC. It means that there is no significant improvement in MRC comparing with EGC. The reason is that the combining effect in very low SNR regime does not offer significant improvement in output SNR. However we can find that PDF itself gets improved if the SNR gets higher, but unfortunately it doesn't directly contribute to P_{D} because it has already converged on unity.

Second, the performance of the energy detector were investigated under different values of noise uncertainty level, ρ, where $\rho = 0$ and $\rho = .5$. $\rho = 0$ means that the noise variance is perfectly known, $\rho = .5$ dB means that the noise variance estimation has error of $.5$ dB in maximum [25], [26]. The result shows that the energy detector outperforms that of the cyclic detector as long as the noise variance is perfectly known. However, the performance of the energy detector is significant degraded under the noise uncertainty. For example, the energy detector cannot detect the signal when the SNR is below -8 dB for the noise variance error of $.5$ dB otherwise the performance of the cyclic detectors are better than that of the energy detector. It is obvious that the robustness against the noise uncertainty of the cyclic detectors is a big advantage over the energy detection in practical situation.

The detection probabilities of the multiple cyclic detector which is based on MRC is depicted in Fig. 7. The result shows that the performance is improved as long as N_α is increased. However, there was quite small difference in cases of $N_\alpha > 7$. Thus, we can conclude that $N_\alpha = 7$ is sufficient to consider for the multiple cyclic detector. For the sensing time of 11.34 ms (10 OFDM symbols), it is seen that the multiple cyclic frequency detector can detect the signal at SNR of -14 dB

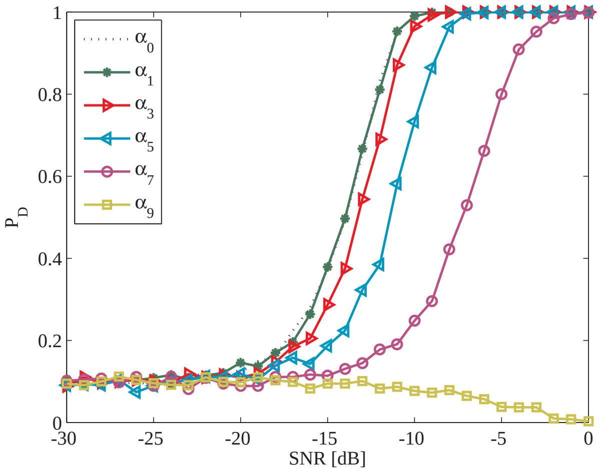

Fig. 5. Detection probabilities of a single cyclic detector for each cyclic frequency with 10 OFDM symbols and $P_{\mathrm{FA}} = 10\%$ where the results are obtained from simulations over $1,000$ trials.

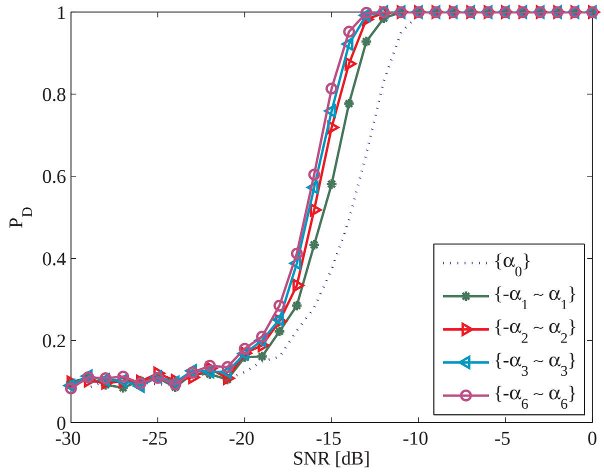

Fig. 7. Detection probabilities of the MRC detector combined by each cyclic frequency set with 10 OFDM symbols and $P_{\mathrm{FA}} = 10\%$ where the results are obtained from simulations over $1,000$ trials.

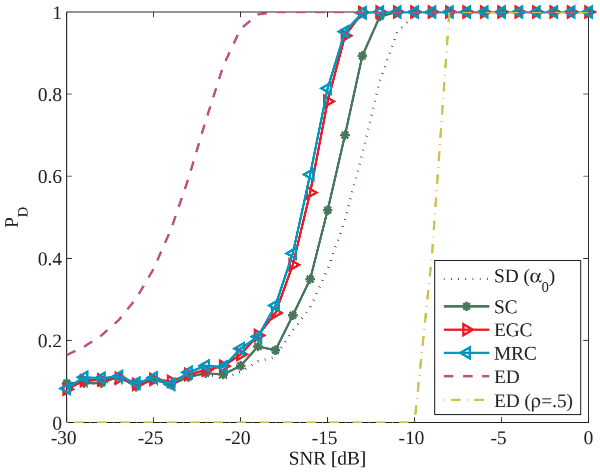

Fig. 6. Comparison between the energy detector (ED) and the single cyclic detector (SD), multiple cyclic detectors of SC, EGC and MRC; It shows the probability of detection P_{D} vs. SNR obtained over $1,000$ trials and the probability of false alarm P_{FA} of 10%. The performance of the energy detector was evaluated for $\rho = 0$ dB and $\rho = 0.5$ dB.

($P_{\mathrm{D}} > 90\%$).

V. Conclusion

In this paper, three combining methods including SC, EG and MRC in cyclic detector for primary OFDM signal working in an AWGN channel have been examined. Herein, we introduced the reduced scheme of Dandawate's algorithm that employs single CAF at time lag $l = N_u$. We also proposed maximum ratio combining method based on transmit signal characteristic, which is easily calculated from the closed-form solution. From the P_{D} evaluation results, we found that MRC had the best performance even if it was just slightly improved over EGC. It is also seen that the combining effect in very low SNR regime did not offer significant improvement in output SNR. For the sensing time of 11.34 ms (10 OFDM symbols), the multiple cyclic detector could detect the signal at SNR of around -14 dB. This work is being expected to be an application to IEEE 802.22 WRAN system.

References

[1] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communication," *IEEE J. Selected Areas in Commun.*, vol. 23, no. 2, pp. 201–221, Feb. 2005.

[2] J. Mitola, "Cognitive radio: Making software radio more personal," *IEEE Personal Comm.*, Vol. 6, No. 4, pp. 13–18,Aug. 1999.

[3] IEEE 802.22 Working Group on Wireless Regional Area Network, http://www.ieee802.org/22/.

[4] C. Cordeiro, K. Challapali, D. Birru, S. Shankar, "IEEE 802.22: the first worldwide wireless standard based on cognitive radio," *2005 First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2005)*, pp. 328–337, Nov. 2005.

[5] H. Tang "Some Physical Layer Issues of Wide-band Cognitive Radio Systems," in proc. *2005 First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2005)*, pp. 151–159, Nov. 2005.

[6] A. Gardner, Cyclostationarity in Communications and Signal Processing, *IEEE Press*, 1994.

[7] A. Gardner, C.M. Spooner, "Signal Interception: A Unifying Theoretical Framework for Feature Detection," *IEEE Trans. Commun.*, vol. 36, no. 8, pp. 897–906, Aug. 1988.

[8] A. Gardner, C.M. Spooner, "Signal Interception: Performance Advantages of Cyclic-Feature Detectors," *IEEE Trans. Commun.*, vol. 40, no. 1, pp. 149–159, Jan. 1992.

[9] V. Dandawate, B. Giannakis, "Statistical Tests for Presence of Cyclostationarity," *IEEE Trans. Signal Processing*, vol. 42, no. 9, pp. 2355–2369, Sept. 1994.

[10] M. Oner and F. Jondral, "Cyclostationarity Based Air Interface Recognition For Software Radio Systems," *Radio and Wireless Conference*, pp. 263 - 266, Sept. 2004.

[11] M. Oner and F. Jondral, "Extracting the Channel Allocation Information in a Spectrum Pooling System Exploiting Cyclostationarity," in Proc. *15th IEEE International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, pp. 551–555, Sept. 2004.

[12] J. Lunden, V. Koivunen, A. Huttunen, H. Vincent Poor, "Spectrum sensing in Cognitive Radios Based on Multiple Cyclic Frequencies," in Proc. *2nd International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom)*, pp. 37–43, Aug. 2007.

[13] J. Lunden, V. Koivunen, A. Huttunen, H. Vincent Poor, "Collaborative Cyclostationary Spectrum Sensing for Cognitive Radio Systems," *IEEE Trans. Signal Processing*, vol. 57, no. 11, pp. 4182–4195, Nov. 2009.

[14] N. Khambekar, L. Dong, V. Chaudhary, "Utilizing OFDM Guard Interval for Spectrum Sensing," in Proc. *IEEE WCNC 2007*, pp.38–42, March 2007.

[15] K. Kim, I. Akbar, K. Bae, U. Jung-sun, C. Spooner, J. Reed, "Cyclostationary Approaches to Signal Detection and Classification in Cognitive Radio," in Proc. *International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2007)*, pp.212–215, April 2007.

[16] P. Sutton, J. Lotze, K. Nolan, L. Doyle, "Cyclostationary Signature Detection in Multipath Rayleigh Fading Environments", in Proc. *2nd International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM 2007)*, August, 2007.

[17] Z. Ye, J. Grosspietsch, G. Memik, "Spectrum Sensing Using Cyclostationary Spectrum Density for Cognitive Radios," in Proc. *IEEE Workshop on Signal Processing Systems 2007*, pp.1–6, Oct. 2007

[18] J. Lee, J. Yoon, J. Kim, "A New Spectral Correlation Approach to Spectrum Sensing for 802.22 WRAN System," in Proc. *Intelligent Pervasive Computing 2007*, pp.101–104, Oct. 2007.

[19] H. Rowe, Signals and Noise in Communication Systems. London, U.K.: Van Nostrand, 1965.

[20] "Transmission System for Digital Terrestrial Television Broadcasting," ARIB STD-B31 Version 1.5, *Association of Radio Industries and Businesses*, 2003 .

[21] F. Digham, M. Alouini, M. Simon, "On Energy Detection of Unknown Signals Over Fading Channels," IEEE Trans. Commun., vol. 55, no. 1, pp. 21-24, Jun. 2007.

[22] J. Ma and Y. G. Li, "Soft Combination and Detection for Cooperative Spectrum Sensing in Cognitive Radio Networks," in Proc. *IEEE GLOBECOM 2007*, pp. 3139-3143, Nov. 2007.

[23] Z. Quan, S. Cui and A. Sayed, "An Optimal Strategy for Cooperative Spectrum Sensing in Cognitive Radio Networks," in proc. *IEEE GLOBECOM'07*, 2007.

[24] Batu K. Chalise and Luc Vandendorpe, "A Multiuser MIMO Transmit Beamformer Based on the Statistics of the Signal-to-Leakage Ratio," *EURASIP J. Wireless Commun. Networking*, vol. 2009, Article ID 679430, 2009.

[25] S. Shellhammer, "Numerical Spectrum Sensing Requirements," IEEE 802.22-06-0088r0, June. 2006.

[26] R. Tandra, A. Sahai, "SNR Walls for Signal Detection," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 4–17, Feb. 2008.