BER Performance of UWB Communications with Matched Filter and Correlation Receivers

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***Abstract*— In this paper, the bit error rate (BER) performance of ultra wideband (UWB) communications with matched filter and correlation receivers are analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which is satisfied the UWB signal definition and Federal Communications Commission (FCC) indoor and outdoor limit spectral masks, are used as the transmitted UWB signal. The complex form of Friis' transmission formula is considered as the UWB free space channel. Therefore, the distortion effects caused from the channel are included. The BER performance of each waveform is shown and compared. The results are discussed in the conclusion.**

I. Introduction

An ultra wideband (UWB) radio technology has become an important topic for microwave communication because of its low cost and low power consumption potentials [1]-[3]. The UWB differs from the convectional narrow band radio frequency (RF) and spread spectrum (SS) technologies. The UWB uses an extremely wideband of RF spectrum to transmit the data with very short pulses and power spectral density (PSD) in the range of ultra wide frequency spectrum instead of using narrow carrier frequency in traditional RF technologies. The UWB is a unique and new usage of recently legalized frequency spectrum. The UWB technology is specified the frequencies ranging from 3.1 GHz to 10.6 GHz by Federal Communications Commission (FCC) [4]. The FCC defined the UWB signal as those, which have a fractional bandwidth equal or greater than 0.20, or occupied bandwidth equal or greater than 500 MHz.

The Friis' transmission formula [5] is widely used to calculate the free space path loss for narrow band communications. After that, the complex form of Friis' transmission formula is developed for UWB communications [6]-[8]. The matched filter and correlation receivers are used as the UWB receivers [9]-[12]. Although, the performances of UWB communications are analyzed [13], [14], there are no considerations about the FCC regulation of UWB signal and distortion of UWB signal caused by channel. After that, the rectangular waveform distorted by UWB free space channel is used to derive the theoretical BER performance [15]. However, there are no considerations about other causal waveforms.

In this paper, the BER performance of UWB communications with matched filter and correlation receivers are analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which is satisfied the UWB signal definition and FCC indoor and outdoor limit spectral masks [16], are used as the transmitted UWB signal. The complex form of Friis' transmission formula is considered as the UWB free space channel. The spectral density of received signal considering the distortion caused by free space channel is evaluated. At the receiver, the matched filter and correlation receivers with frequency transfer functions, which is satisfied the constant noise power condition between the input and output, are used. The BER performance of each waveform is shown and compared.

This paper is organized as follows. Sections 2 and 3, the UWB waveform models and analysis of BER performance are briefly discussed, respectively. Next the analysis results are illustrated in Sec. 4. Finally, the conclusions are discussed in Sec. 5.

II. UWB Waveform Models

For UWB waveforms, the rectangular passband, modulated rectangular and modulated Gaussian waveforms are considered as the UWB transmitted waveform (v_{t}) in time domain and its spectral density (V_{t}) in frequency domain. These waveforms can satisfy the FCC definition of UWB signal and FCC spectral masks for indoor and outdoor limits. The parameters obtained from maximum bandwidth, amplitude and average power optimizations, which is proposed in [16], are used.

A. Rectangular Passband Waveform

The rectangular passband transmitted waveform in time domain and its spectral density function are given by

$$v_{\mathrm{t}}(t) = \frac{A}{f_{\mathrm{b}}}[f_{\mathrm{H}}\mathrm{sinc}(2f_{\mathrm{H}}t) - f_{\mathrm{L}}\mathrm{sinc}(2f_{\mathrm{L}}t)], \quad (1)$$

$$V_{\mathrm{t}}(f) = \begin{cases} \frac{A}{2f_{\mathrm{b}}} & ||f| - f_{\mathrm{c}}| \le \frac{f_{\mathrm{b}}}{2} \\ 0 & ||f| - f_{\mathrm{c}}| > \frac{f_{\mathrm{b}}}{2} \end{cases}, \quad (2)$$

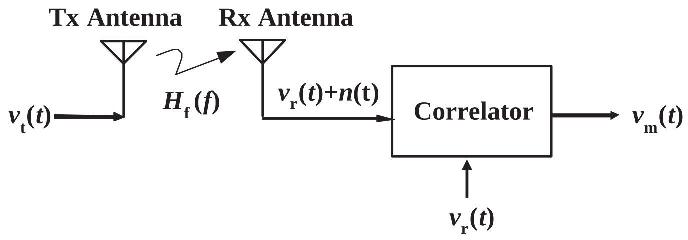

Fig. 1. Block diagram of matched filter receiver system.

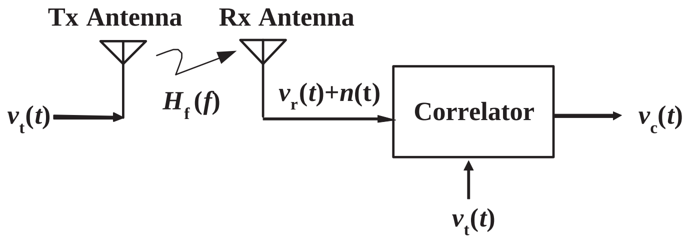

Fig. 2. Block diagram of correlation receiver system.

where A is the maximum amplitude, f_b is the occupied bandwidth, f_c is the center frequency, $f_\mathrm{L} = f_\mathrm{c} - f_\mathrm{b}/2$ and $f_\mathrm{H} = f_\mathrm{c} + f_\mathrm{b}/2$ are the minimum and maximum frequencies.

B. Modulated Rectangular Waveform

The modulated rectangular transmitted waveform in time domain and its spectral density function are given by

$$v_\mathrm{t}(t) = \begin{cases} A\sin(2\pi f_\mathrm{c} t) & |t| \le \frac{t_\mathrm{b}}{2} \\ 0 & |t| > \frac{t_\mathrm{b}}{2} \end{cases}, \qquad (3)$$

$$V_\mathrm{t}(f) = \frac{A t_\mathrm{b}}{j2} \left\{ \begin{array}{c} \mathrm{sinc}[t_\mathrm{b}(f-f_\mathrm{c})] \\ -\mathrm{sinc}[t_\mathrm{b}(f+f_\mathrm{c})] \end{array} \right\}, \qquad (4)$$

where A is the maximum amplitude, f_c is the carrier frequency and t_b is the pulse width of waveform.

C. Modulated Gaussian Waveform

The modulated Gaussian transmitted waveform in time domain and its spectral density function are given by

$$v_\mathrm{t}(t) = Ae^{-(t/d)^2}\sin(2\pi f_\mathrm{c} t), \qquad (5)$$

$$V_\mathrm{t}(f) = \frac{Ad\sqrt{\pi}}{j2} \left[\begin{array}{c} e^{-\pi^2 d^2 (f-f_\mathrm{c})^2} \\ -e^{-\pi^2 d^2 (f+f_\mathrm{c})^2} \end{array} \right], \qquad (6)$$

where A is the maximum amplitude of envelope waveform, f_c is the carrier frequency and d is the $1/e$ characteristic decay time.

III. Analysis of BER Performance

In this section, the analysis of BER performance is theoretically discussed. The UWB waveform models discussed in Sec. 2 are used as the UWB transmitted waveforms. For UWB free space channel, the complex form of Friis' transmission formula is used [6]-[8]. The transmitting (Tx) and receiving (Rx) antennas are considered to have one constant gains. The frequency transfer function of free space channel H_f can be written as

$$H_\mathrm{f}(f,d) = \frac{c}{4\pi|f|d} e^{-j2\pi f d/c}, \qquad (7)$$

where d is the transmitter-receiver (T-R) separation distance and c is the velocity of light. This equation is satisfied for both positive and negative frequencies as it satisfies the causality.

The spectral density of UWB received signal V_r is calculated by using multiplication between H_f and V_t, which can be written as

$$V_\mathrm{r}(f,d) = H_\mathrm{f}(f,d) \cdot V_\mathrm{t}(f). \qquad (8)$$

This spectral density of UWB received signal includes the distortion effect caused by UWB free space channel.

The matched filter and correlation receivers with frequency transfer functions satisfied constant noise power condition between input and output are considered. For matched filter receiver or optimum correlation receiver, the spectral density of template signal is the complex conjugate of V_r. The block diagram of matched filter receiver system is shown in Fig. 1. For correlation or transmitted template signal receiver, the spectral density of template signal is the complex conjugate of V_t. Figure 2 shows the block diagram of correlation receiver system. The receiver gains of both receivers are satisfied the constant noise power condition. Therefore, the frequency transfer functions of matched filter and correlation receivers, (H_m and H_c), can be written as

$$H_\mathrm{m}(f,d) = \frac{\sqrt{2f_\mathrm{b}}}{\sqrt{\int_{-\infty}^{\infty} |V_\mathrm{r}(f,d)|^2 df}} V_\mathrm{r}^*(f,d), \qquad (9)$$

$$H_\mathrm{c}(f,d) = \frac{\sqrt{2f_\mathrm{b}}}{\sqrt{\int_{-\infty}^{\infty} |V_\mathrm{t}(f,d)|^2 df}} V_\mathrm{t}^*(f,d), \qquad (10)$$

where * is the complex conjugate operator.

The spectral densities of output signal from matched filter and correlation receivers, (V_m and V_c), can be written as

$$V_\mathrm{m}(f,d) = H_\mathrm{m}(f,d) \cdot V_\mathrm{r}(f,d), \qquad (11)$$

$$V_\mathrm{c}(f,d) = H_\mathrm{c}(f,d) \cdot V_\mathrm{r}(f,d). \qquad (12)$$

The signal-to-noise ratio (SNR) gain of these receivers are defined as the ratio between average power of received signal at receiver input and that at receiver output. The SNR gains of matched filter and correlation receivers, G_m and G_c can be respectively written as

$$G_\mathrm{m} = \frac{\int_{-\infty}^{\infty} |V_\mathrm{m}(f,d)|^2 df}{\int_{-\infty}^{\infty} |V_\mathrm{r}(f,d)|^2 df}, \qquad (13)$$

$$G_\mathrm{c} = \frac{\int_{-\infty}^{\infty} |V_\mathrm{c}(f,d)|^2 df}{\int_{-\infty}^{\infty} |V_\mathrm{r}(f,d)|^2 df}. \qquad (14)$$

The efficiency of receiver is considered by using correlation coefficient between received and template signals. The correlation coefficients of matched filter and correlation receivers, C_m

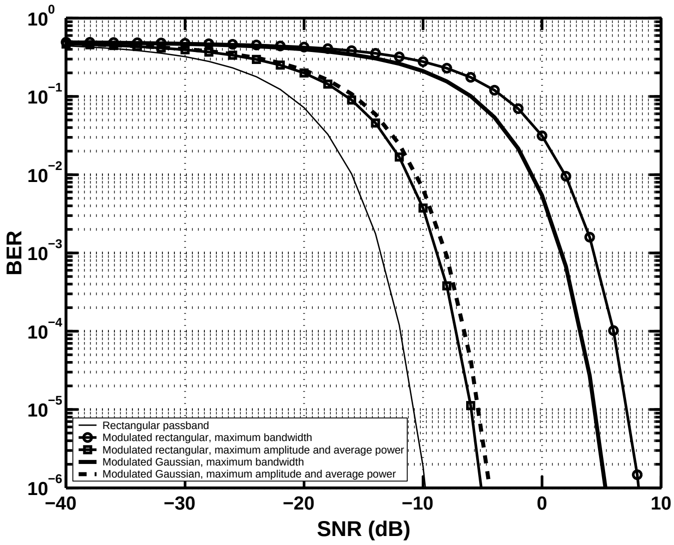

Fig. 3. BER performance of matched filter receiver for UWB waveforms satisfied FCC indoor limit spectral mask.

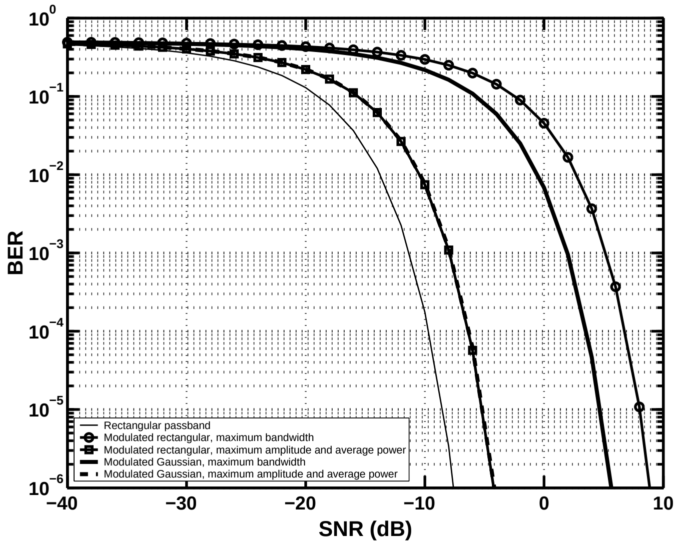

Fig. 4. BER performance of correlation receiver for UWB waveforms satisfied FCC indoor limit spectral mask.

and $C_{\rm c}$ can be respectively written as

$$C_{\rm m} = \frac{\max|\int_{-\infty}^{\infty} V_{\rm m}(f,d)e^{j2\pi ft}df|}{\sqrt{\int_{-\infty}^{\infty}|V_{\rm r}(f,d)|^2df\cdot\int_{-\infty}^{\infty}|H_{\rm m}(f,d)|^2df}}, \quad (15)$$

$$C_{\rm c} = \frac{\max|\int_{-\infty}^{\infty} V_{\rm c}(f,d)e^{j2\pi ft}df|}{\sqrt{\int_{-\infty}^{\infty}|V_{\rm r}(f,d)|^2df\cdot\int_{-\infty}^{\infty}|H_{\rm c}(f,d)|^2df}}, \quad (16)$$

The UWB modulation schemes can be classified in to the antipodal modulation scheme such as binary pulse amplitude modulation (BPAM) and orthogonal modulation scheme such as on-off keying (OOK) and pulse position modulation (PPM) with modulation index of $\delta = 1$ [17]. In this paper, only BER performance of antipodal modulation scheme is considered. For orthogonal modulation scheme, the BER performance can be evaluated in the same way as shown in [15], [18]. The BER performance of matched filter and correlation receivers, $B_{\rm m}$ and $B_{\rm c}$, in additive white Gaussian noise (AWGN) for antipodal modulation scheme can be respectively written as

$$B_{\rm m} = Q\left(\sqrt{\frac{2C_{\rm m}G_{\rm m}f_{\rm b}S}{B_{\rm r}N}}\right), \quad (17)$$

$$B_{\rm c} = Q\left(\sqrt{\frac{2C_{\rm c}G_{\rm c}f_{\rm b}S}{B_{\rm r}N}}\right), \quad (18)$$

where $B_{\rm r}$ is the bit rate, S/N is the SNR at input of receiver and

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt, \quad x \geq 0.$$

The SNR is normalized by average power of rectangular passband waveform, which is the theoretical maximum average power of UWB signal, for considering FCC spectral masks. Moreover, the distortion effect caused from UWB channel, occupied bandwidth and bit rate are also included to analyze the BER performance.

IV. Analysis Results

In this section, the analysis results of BER performance are shown. The BER performance of transmitted waveforms, which are the rectangular passband, modulated rectangular and modulated Gaussian waveforms, are analyzed. The T-R separation distance is set to be 10 m, while the bit rate is set to be 110 Mbps. These parameters are based on the IEEE 802.15.3a [19]. The parameters of each waveform, which obtained from the maximum bandwidth, amplitude and average power optimizations satisfied FCC spectral masks for indoor and outdoor limits [15], are used.

Figures 3 and 4 show the BER performances of matched filter and correlation receivers for UWB waveforms satisfied FCC indoor limit spectral mask, respectively. The BER performances of rectangular passband waveforms are lowest for both matched filter and correlation receivers. The BER performances of maximum amplitude and average power optimizations are less than that of maximum bandwidth optimizations. That is because the maximum bandwidth optimizations are much less average power and slightly more bandwidth compared with maximum amplitude and average power optimizations [16]. For comparison between modulated rectangular and Gaussian waveforms, the BER performance of modulated rectangular and Gaussian waveforms with maximum amplitude and average power optimizations are almost the same and are better than the others.

The BER performances of matched filter and correlation receivers for UWB waveforms satisfied FCC outdoor limit spectral mask are shown in Figs. 5 and 6, respectively. The BER performances of rectangular passband waveforms are also lowest for both matched filter and correlation receivers. For

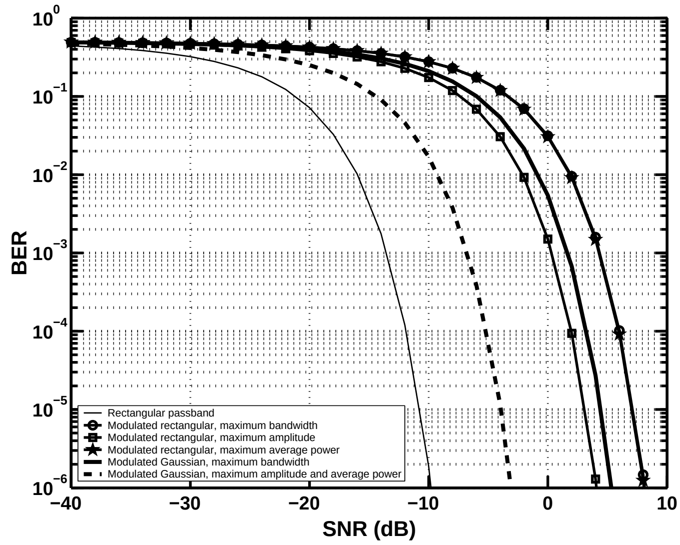

Fig. 5. BER performance of matched filter receiver for UWB waveforms satisfied FCC outdoor limit spectral mask.

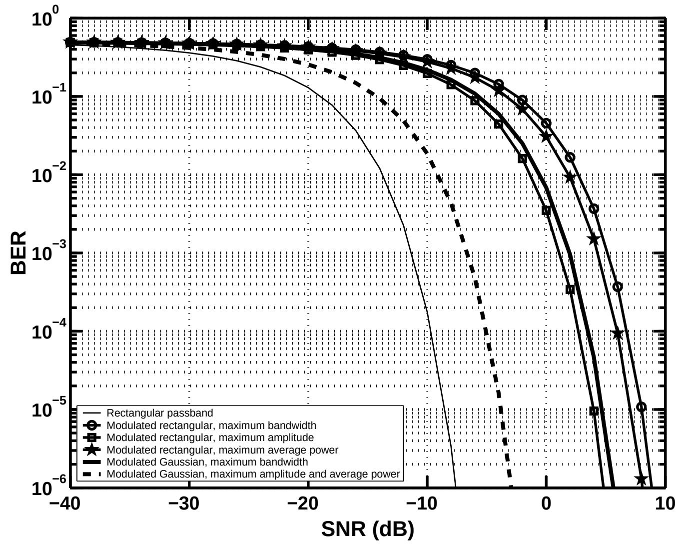

Fig. 6. BER performance of correlation receiver for UWB waveforms satisfied FCC outdoor limit spectral mask.

comparison between modulated rectangular and Gaussian waveforms, the BER performance of modulated Gaussian waveform with maximum amplitude and average power optimizations are clearly better than the others.

V. Conclusion

In this paper, the BER performance of UWB communications with matched filter and correlation receivers are analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which is satisfied the UWB signal definition and FCC indoor and outdoor limit spectral masks, are used as the transmitted UWB signal. From the results, the BER performance of rectangular passband waveform with matched filter receiver is the ideal theoretical bound, which is the minimum BER performance case. The modulated rectangular and Gaussian waveforms with maximum amplitude and average power optimizations are appropriate for FCC indoor limit spectral mask, while the modulated Gaussian waveform with maximum amplitude and average power optimizations is appropriate for FCC outdoor limit spectral mask.

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