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Application of Fourth-Order Cumulants to Delay Time and DOA Estimation of Multiple Delayed Waves by the MMP Method

Yuichi Shimamura*

Department of Electronic Engineering, Graduate School of Science and Engineering,
Saitama University, Urawa, Japan 338–8570

Kiyomichi Araki

Department of Computer Engineering, Tokyo Institute of Technology, Tokyo, Japan 152-8552

Jun-ichi Takada

International Cooperative Center for Science and Technology, Tokyo Institute of Technology, Tokyo, Japan 152-8552

SUMMARY

This paper proposes an estimation method for the delay time, the direction of arrival, and the signal strength of multiple delayed waves, which are important parameters characterizing the multiple delay characteristics of mobile radio communication systems. The proposed method is an estimation method in which the superresolution and higher-order statistics are used as the guiding principle. The frequency characteristics of the multiple propagation path environment are observed by a linear array antenna, and estimated by the modified matrix pencil (MMP) technique, by generating the virtual correlation matrix from the result of processing of the fourth-order cumulants. The method can estimate at least twice as many delayed waves as when the second-order moment is used, if the number of observation points on the frequency axis and the number of antenna elements are the same. A method is also proposed in which the error in the generation of the virtual correlation matrix is reduced. The effectiveness of the proposed method is demonstrated by computer simulation. © 1999 Scripta Technica, Electron Comm Jpn Pt 1, 82(12): 30–39, 1999

Key words: Multiple delayed waves; estimation of delay time; estimation of direction of arrival; superresolution method; higher-order statistics.

1. Introduction

At present, mobile communication services are widely utilized as means of personal communications, and the demands are still increasing rapidly. As another aspect, the function of transmitting multimedia information is considered necessary in nonverbal services, for which increasing demand is expected. From such a viewpoint, in the next

* Presently with NTT.

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generation mobile radio communication systems must drastically improve their spectral efficiency and transmission rate by techniques such as separation of the paths from multiple mobile stations using the same frequency at the same time [1], and elimination of intersymbol interference. In order to meet these requirements, adequate identification or estimation of the multiple propagation paths between the mobile station and the base station, that is, estimation of the parameters characterizing the multiple delay characteristics, is crucial.

In order to realize such an estimation procedure, the superresolution technique and higher-order statistics are utilized as the guiding principle in the method proposed in this paper. Typical methods in the superresolution technique are MUSIC [2] and ESPRIT [3]. The method achieves higher resolution than traditional methods such as the FFT. As a delay estimation method belonging to the superresolution technique, the MMP (modified matrix pencil) method has been presented [4]. The MMP method is a superresolution method that can realize high estimation accuracy without using the peak search process as in MUSIC.

Furthermore, only the second-order statistics such as the mean and variance are usually utilized in traditional signal processing. The application of higher-order statistics to the signal processing technique is also being considered at present [5, 6]. By utilizing higher-order statistics in the superresolution method, it is reported that a method for estimating the direction of arrival, with performance exceeding the traditional method [7], can be developed. With this as background, we now propose a method of estimation of the delay time, the direction of arrival, and the signal strength, by the MMP method utilizing fourth-order cumulants [8, 9].

As the first step, the frequency characteristics of the multipath propagation environment in mobile radio communication is formulated, and the fourth-order cumulants used as higher-order statistics are described. Then, the virtual correlation matrix is generated using the fourth-order cumulants, and a method that estimates the delay time by the MMP technique is presented. By applying the delay time estimation process, an estimation method is proposed for the direction of arrival, the correspondence between the delay time and the direction of arrival, and the signal strength. Lastly, the performance of the proposed method is evaluated by a computer simulation.

2. Multiple Propagation Paths

2.1. Multipath propagation environment in mobile communications

In a mobile radio communication system, multiple propagation paths are formed between the mobile station and the base station as a result of reflection and refraction by the artificial or natural reflectors around the base and mobile stations. In other words, the wave received at the base station is composed of waves arriving with various delay times and directions of arrival. In addition, not only reflected waves with a constant deterministic component within the interval of observation, but also scattered waves with stochastic fluctuations produced by scattering around the mobile station, are combined with the above multiple propagation paths. With the movement of the mobile station, rapidly fluctuating fading is produced by the scattered waves.

Such a multiple propagation path environment is represented by a model called the GWSSUS (Gaussian wide-sense stationary uncorrelated scattering) channel [4]. In a time interval corresponding to several tens of wavelengths, for which the model is considered valid, there is little fluctuation in the delay time or the direction of arrival of the received wave. Since the scattering processes on the propagation paths are independent, it is assumed that the fluctuations of the amplitudes and phases of the arriving waves are mutually uncorrelated [4].

2.2. Formulation of frequency characteristics of multiple propagation path environment

The frequency characteristics of the multiple propagation path environment are formulated as follows. Consider a straight-line array antenna of L elements with spacing Δx, as shown in Fig. 1. Let the frequency characteristics of the multiple propagation path environment be measured at N points at equal intervals $\Delta\omega$ on the frequency axis, and M points at equal intervals Δt on the time axis. Then, the frequency characteristics $H(\mu, l, i)$ $(=H(w, x, t))$ of the multiple propagation path environment at the μ-th frequency and the i-th time are formulated as follows. We set $\omega = \omega_0 + \Delta\omega \cdot \mu$, $x = \Delta x \cdot l$, $t = \Delta t \cdot i$. In the above, ω_0 is the angular frequency of the carrier wave. It is assumed that the antenna elements have the same characteristics without directivity, and that there is no interaction between the antenna elements.

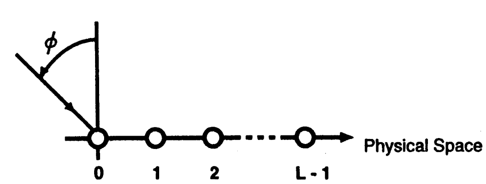

Fig. 1. Linear array antenna.

Additive white Gaussian noise $n(\mu, l, i)$ is assumed as the observation noise component. Then, the result of measurement is

$$\begin{aligned}
&\widehat{H}(\mu,l,i)\\
&= H(\mu,l,i)+n(\mu,l,i)\\
&= \sum_{\nu=1}^{p} \underbrace{\left[A_\nu e^{j\omega_{d\nu}\Delta t\cdot i} + a_\nu^{(s)}(i)\right]}_{a_\nu(i)}\\
&\quad \times e^{-j\{\Delta\omega\cdot\mu\cdot\tau_\nu+(\omega_0+\Delta\omega\cdot\mu)\Delta x\cdot l/c\cdot\sin\phi_\nu\}}\\
&\quad + n(\mu,l,i)\\
&\mu = \frac{N-1}{2}, \frac{N-1}{2}-1,\cdots,-\frac{N-1}{2}\\
&l = 0,1,\cdots,L-1\ ,\ i=0,1,\cdots,M-1
\end{aligned} \tag{1}$$

The first term in brackets in Eq. (1) is the deterministic reflected component. Let the complex amplitude of the ν-th reflected wave be A_ν and the Doppler angular frequency be $\omega_{d\nu}$. The second term $a_\nu^{(s)}(i)$ is the scattered wave component with Gaussian stochastic fluctuation. In other words, the Rice propagation path environment is assumed. In the above, p is the total number of delayed waves (the number of propagation paths), $a_\nu(i)$ is the complex amplitude of the delayed wave, τ_ν is the delay time of the delayed wave, ϕ_ν is the direction of arrival of the delayed wave, and c is the speed of light.

Assuming further that the frequency bandwidth of measurement is sufficiently narrow compared to the carrier frequency, the following approximation can be applied

$$\begin{aligned}
&\widehat{H}(\mu,l,i)\\
&\simeq \sum_{\nu=1}^{p} a_\nu(i) e^{-j(\Delta\omega\cdot\mu\cdot\tau_\nu+\omega_0\Delta x\cdot l/c\cdot\sin\phi_\nu)}\\
&\quad + n(\mu,l,i)\\
&= \sum_{\nu=1}^{p} a_\nu(i) z_{\tau_\nu}^{\mu} z_{\phi_\nu}^{l} + n(\mu,l,i)
\end{aligned} \tag{2}$$

where

$$z_{\tau_\nu} = e^{-j\Delta\omega\tau_\nu} \tag{3}$$

$$z_{\phi_\nu} = e^{-j\omega_0\Delta x/c\cdot\sin\phi_\nu} \tag{4}$$

It is assumed that all mobile stations are sending signals with the same amplitude at all frequencies that are measured. For the details of the generation of such signals and the method of frequency analysis, see Ref. 4.

3. Cumulants

The statistics of third and higher order are called higher-order statistics (HOS), and their applications to the signal processing technique have been investigated [5, 6]. Dogan and Mendel [7] presented a method to further improve the resolution by utilizing high-level phase information and the Gaussian noise suppression effect of the fourth-order cumulant, which is one of the higher-order statistics, in superresolution methods such as MUSIC [2] and ESPRIT [3]. In the following, the properties of the statistics called cumulants are described.

3.1. Fourth-order cumulant

For the stochastic variable vector $\boldsymbol{x}_i$ with mean 0, the fourth-order cumulant with 0 as the center is defined as follows, where E[ ] represents the ensemble average:

$$\begin{aligned}
&\mathrm{cum}[x_1(t),x_2(t),x_3(t),x_4(t)]\\
&= \mathrm{E}[x_1(t)x_2(t)x_3(t)x_4(t)]\\
&\quad - \mathrm{E}[x_1(t)x_2(t)]\cdot\mathrm{E}[x_3(t)x_4(t)]\\
&\quad - \mathrm{E}[x_1(t)x_3(t)]\cdot\mathrm{E}[x_2(t)x_4(t)]\\
&\quad - \mathrm{E}[x_1(t)x_4(t)]\cdot\mathrm{E}[x_2(t)x_3(t)]
\end{aligned} \tag{5}$$

Definition (5) can also be applied to the complex stochastic variable [6].

3.2. Properties of cumulants

Some important properties of the cumulants are now described. Those properties also apply to the case of the complex stochastic variables [6].

(1) When a stochastic variable is multiplied by a (complex) constant, the cumulant itself is multiplied by that constant. Let α_i be a constant and let x_i be a stochastic variable. Then,

$$\begin{aligned}
&\mathrm{cum}[\alpha_1 x_1, \alpha_2 x_2, \cdots, \alpha_n x_n]\\
&= \left(\prod_{i=1}^{n}\alpha_i\right)\mathrm{cum}[x_1,x_2,\cdots,x_n]
\end{aligned} \tag{6}$$

(2) The cumulant of the sum of independent stochastic variables is equal to the sum of the respective cumulants of those variables. When the stochastic variables x_i and y_i are mutually independent, we have

$$\begin{aligned}
&\mathrm{cum}[x_1+y_1, x_2+y_2, \cdots, x_n+y_n]\\
&= \mathrm{cum}[x_1,x_2,\cdots,x_n] + \mathrm{cum}[y_1,y_2,\cdots,y_n]
\end{aligned} \tag{7}$$

(3) The third- or higher-order cumulant suppresses the Gaussian signal. Let z_i be a Gaussian stochastic variable and let $n \geqq 3$. Then,

$$\mathrm{cum}[z_1, z_2, \cdots, z_n] = 0 \tag{8}$$

4. Delay Time Estimation by MMP Using Fourth-Order Cumulant

This paper proposes the following delay time estimation method for multiple delayed waves. The virtual correlation matrix is generated from the result obtained by processing the frequency characteristics of the multiple propagation paths using the fourth-order cumulant. The MMP technique is applied to the virtual correlation matrix to obtain the estimation [8].

4.1. Processing of frequency characteristics by fourth-order cumulant

Consider a single antenna element (let l = 0, for example). The measured frequency characteristic is defined as follows:

$$\widehat{H}_\mu \triangleq \widehat{H}(\mu, 0, i) = H_\mu + n_\mu \tag{9}$$

Assume that the above frequency characteristic is observed at three points on the frequency axis, from the center frequency $\mu = 0$ to the second adjacent point $\mu = 2$. The following fourth-order cumulant is calculated from three observed values $\hat{H}_0^*$, $\hat{H}_1$, $\hat{H}_2$ as shown in Fig. 2 (the asterisk indicates the complex conjugate):

$$\begin{aligned}
&\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_1, \widehat{H}_0^*, \widehat{H}_2] \\
&\quad = \mathrm{cum}[H_0^*, H_1, H_0^*, H_2] \\
&\qquad + \mathrm{cum}[n_0^*, n_1, n_0^*, n_2]
\end{aligned} \tag{10}$$

In the above calculation, property (2) of the cumulant in Section 3.2 is used, since H and n are statistically independent.

We also have

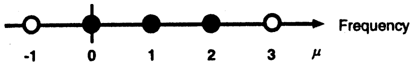

Fig. 2. Measurement in the frequency domain.

$$\begin{aligned}
&\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_1, \widehat{H}_0^*, \widehat{H}_2] \\
&\quad = \sum_{\nu=1}^{p} \mathrm{cum}[a_\nu^*(i), a_\nu(i) z_{\tau_\nu}, a_\nu^*(i), a_\nu(i) z_{\tau_\nu}^2] \\
&\quad = \sum_{\nu=1}^{p} \mathrm{cum}[A_\nu^* e^{-j\omega_{d\nu}\Delta t \cdot i}, A_\nu e^{j\omega_{d\nu}\Delta t \cdot i}, \\
&\qquad A_\nu^* e^{-j\omega_{d\nu}\Delta t \cdot i}, A_\nu e^{j\omega_{d\nu}\Delta t \cdot i}] z_{\tau_\nu}^3 \\
&\quad = -\sum_{\nu=1}^{p} |A_\nu|^4 z_{\tau_\nu}^3
\end{aligned} \tag{11}$$

In the above calculation, property (3) is applied to the scattered wave component and the observed noise component. The terms corresponding to the scattered wave component and the observed noise component, which have Gaussian statistical properties, are zero, and there remains only the term for the deterministic reflected wave component. Since z_{τ_ν} is a constant, property (1) is also used. Property (2) is used, since it is assumed that there is no correlation among the multiple delayed waves, as was discussed in Section 2.1.

When the second-order moment is used, observation at observation point $\mu = 3$ is required in order to determine the delay term $z_{\tau_\nu}^3$. This is not required when the fourth-order cumulant is used, as is seen from Eq. (11). There is an effect that frequency characteristics which are not actually observed are virtually observed.

4.2. Generation of virtual correlation matrix

The virtual correlation matrix is generated in order to apply the matrix pencil in the MMP method. For two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, the matrix pencil is given by $\boldsymbol{A} - z\boldsymbol{B}$, where z is a complex scalar parameter. The special value of z for which the rank of $\boldsymbol{A} - z\boldsymbol{B}$ is decreased is related to the generalized eigenvalue, as will be discussed later.

Consider the case where the frequency characteristics are observed at three points on the frequency axis, as in Fig. 2. Five fourth-order cumulants are calculated so that the delay times z_{τ_ν} to the power of 0 to 4 will be included. The quality C_n defined as follows:

$$C_n \triangleq -\sum_{\nu=1}^{p} |A_\nu|^4 z_{\tau_\nu}^n \tag{12}$$

The following relation is clear from the derivation of Eq. (11):

$$\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_0, \widehat{H}_0^*, \widehat{H}_0] = C_0 \tag{13}$$

$$\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_1, \widehat{H}_0^*, \widehat{H}_0] = C_1 \tag{14}$$

$$\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_2, \widehat{H}_0^*, \widehat{H}_0] = C_2 \tag{15}$$

$$\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_1, \widehat{H}_0^*, \widehat{H}_2] = C_3 \tag{16}$$

$$\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_2, \widehat{H}_0^*, \widehat{H}_2] = C_4 \tag{17}$$

The virtual correlation matrix $\boldsymbol{S}$ is generated by arranging C_0 to C_4 as elements of a 5×5 matrix. In general, the virtual correlation matrix of size K is

$$\boldsymbol{S} = \begin{bmatrix} C_0 & C_1^* & \cdots & C_{K-1}^* \\ C_1 & C_0 & \cdots & C_{K-2}^* \\ \vdots & \vdots & \ddots & \vdots \\ C_{K-1} & C_{K-2} & \cdots & C_0 \end{bmatrix}$$

$$= -\sum_{\nu=1}^{p} |A_\nu|^4$$

$$\times \begin{bmatrix} 1 & z_{\tau_\nu}^{-1} & \cdots & z_{\tau_\nu}^{-(K-1)} \\ z_{\tau_\nu} & 1 & \cdots & z_{\tau_\nu}^{-(K-2)} \\ \vdots & \vdots & \ddots & \vdots \\ z_{\tau_\nu}^{K-1} & z_{\tau_\nu}^{K-2} & \cdots & 1 \end{bmatrix} \tag{18}$$

The virtual correlation matrix $\boldsymbol{S}$ based on the fourth-order cumulants has a larger size than that of the second-order correlation matrix. This implies that a larger number of arriving waves can be estimated.

4.3. Delay time estimation by MMP

The delay times of the arriving waves are estimated by applying the MMP algorithm [4] to the generated $\boldsymbol{S}$. The MMP algorithm is one of the superresolution methods in which the estimation of the delay time, as well as the direction of arrival, are reduced to the generalized eigenvalue problem. The peak search process is not required as in MUSIC. Another feature is that the matrix with just the rank required for the estimation is used, which helps to improve the accuracy of estimation. There is not much difference between TLS-ESPRIT [3] and MMP [4].

The estimation procedure is as follows. By the use of the fourth-order cumulants, the effect of virtual observation is realized. Thus, the delay times can be estimated for at least twice the number of delayed waves, compared to the traditional method based on the second-order moments.

[Step 1] The virtual correlation matrix is generated from the observed frequency characteristics. Eigenvalue decomposition is applied. The number p of delayed waves is estimated based on the determined rank.

[Step 2] The prediction matrix $\boldsymbol{C}_{(p)}$ is calculated as follows, where the superscript H indicates the complex conjugate transpose:

$$\boldsymbol{C}_{(p)} = \left(\boldsymbol{I} + \frac{\boldsymbol{m}_1 \boldsymbol{m}_1^H}{1 - \boldsymbol{m}_1^H \boldsymbol{m}_1}\right) \cdot \boldsymbol{M}_1^H \boldsymbol{M}_2 \tag{19}$$

[Step 3] Eigenvalue decomposition is applied to the prediction matrix $\boldsymbol{C}_{(p)}$. Then, the eigenvalues agree with the delay terms z_{τ_ν} ($\nu = 1, 2, \ldots, p$).

[Step 4] The delay times z_{τ_ν} of the delayed waves are calculated as follows from the delay terms τ_ν ($\nu = 1, 2, \ldots, p$):

$$\tau_\nu = -\arg\left[z_{\tau_\nu}\right] / \Delta\omega \tag{20}$$

m_1, M_1, M_2 in step 2 are derived from the representation, which is a compression of the eigenvalue decomposition of $\boldsymbol{S}$ to rank p. In other words,

$$\boldsymbol{S} = \boldsymbol{M} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{M}^H = \boldsymbol{M}_{(p)} \cdot \boldsymbol{\Lambda}_{(p)} \cdot \boldsymbol{M}_{(p)}^H \tag{21}$$

The first to the $(K - 1)$-th columns of matrix $\boldsymbol{M}_{(p)}^H$ are written as $\boldsymbol{M}_1^H$, the first column is written as $\boldsymbol{m}_1$, and the second to the K-th column are written as $\boldsymbol{M}_2^H$.

5. Estimation of Direction of Arrival and Signal Strength by MMP Method Using Fourth-Order Cumulants

5.1. Estimation of direction of arrival

In the estimation of the direction of arrival, the direction is observed as the time difference of the arrivals at the antenna elements. The virtual correlation matrix is generated for the direction of arrival z_{ϕ_ν} of the delayed wave, and the direction of arrival is estimated by applying the MMP algorithm. The procedure is the same as in the estimation of the delay time discussed in Section 4.3. Based on the estimated direction of arrival terms z_{ϕ_ν} ($\nu = 1, 2, \ldots, p$), the directions of arrival of the delayed waves ϕ_ν ($\nu = 1, 2, \ldots, p$) are calculated as follows:

$$\phi_\nu = \sin^{-1}\left(-\frac{c \cdot \arg\left[z_{\phi_\nu}\right]}{\omega_0 \Delta x}\right) \tag{22}$$

Using the fourth-order cumulants, the effect of virtual observation of the frequency characteristics is realized for antenna elements which do not actually exist [7]. Compared to the case where the second-order moment is used, the directions of arrival can be estimated for at least twice the number of delayed waves [9].

5.2. Estimation of correspondence between delay time and direction of arrival

The frequency characteristics measured at multiple frequencies in the straight-line array antenna can be represented as follows:

$$\widehat{H}_{\mu,l,i} = \sum_{\nu=1}^{p} a_\nu(i) z_{\tau_\nu}^{\mu} z_{\phi_\nu}^{l} + n(\mu,l,i) \qquad (23)$$

Assume that the above frequency characteristics are observed at the observation points on the oblique line in Fig. 3. Then, the correspondence term $z_{\tau_\nu} z_{\phi_\nu}$ can be estimated by MMP, as in the estimation of the delay times.

In order to establish the correspondence between the delay time τ_ν and the direction of arrival ϕ_ν for multiple delayed waves, the product $z_{\tau_i} z_{\phi_j}$ of the estimated delay term z_{τ_i} $(i = 1, 2, \ldots, p)$ and the estimated direction of arrival term z_{ϕ_j} $(j = 1, 2, \ldots, p)$ is calculated. When the result agrees with one of the estimated correspondence terms $z_{\tau_k} z_{\phi_k}$ $(k = 1, 2, \ldots, p)$, it is estimated that the delay time τ_i corresponds to the direction of arrival ϕ_j [9].

5.3. Estimation of signal strength

The matrix element C_n of the virtual correlation matrix generated for the estimation of the delay time is related to the complex amplitude A_ν of the reflected wave component of the delayed wave as follows. A similar relation also exists in the case of the virtual correlation matrix for the estimation of the direction of arrival:

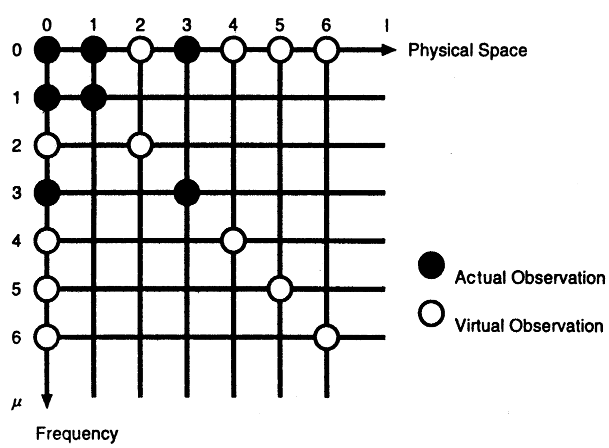

Fig. 3. Observation points.

$$\begin{bmatrix} |A_1|^4 \\ |A_2|^4 \\ \vdots \\ |A_p|^4 \end{bmatrix} = - \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_{\tau_1} & z_{\tau_2} & \cdots & z_{\tau_p} \\ \vdots & \vdots & \ddots & \vdots \\ z_{\tau_1}^{p-1} & z_{\tau_2}^{p-1} & \cdots & z_{\tau_p}^{p-1} \end{bmatrix} \times \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{p-1} \end{bmatrix} \qquad (24)$$

By calculating the square root of $|A_\nu|^4$ estimated by Eq. (24), the signal strength $|A_\nu|^2$ of the reflected wave component can be estimated.

6. Reduction of Error in Virtual Correlation Matrix

The cumulant was originally defined as the ensemble average for the ensemble of the stochastic processes. In the proposed method, however, the average is replaced by the time average for a certain sample function over a finite time.

Because of this situation, when the fourth-order cumulant is calculated from a finite sampled value sequence, the result may deviate greatly from the true value. Due to this deviation, the virtual correlation matrix composed of fourth-order cumulants estimated from a finite number of samples includes an error. The technique used to reduce the computation error in the virtual correlation matrix is shown in the following.

6.1. Reduction of error of virtual correlation matrix

When the observation is made as in Fig. 2, the value of a single fourth-order cumulant is used as the matrix element C_2, as in Eq. (15). When three data for the frequency characteristics are observed, however, several values for the fourth-order cumulant with the same phase information $z_{\tau_\nu}^2$ can be calculated. From such a viewpoint, the arithmetic mean of several fourth-order cumulants with the same phase information is used as the matrix element of the virtual correlation matrix:

$$C_2 = \frac{1}{5}\Big\{\mathrm{cum}[\widehat{H}_0^*, \widehat{H}_2, \widehat{H}_0^*, \widehat{H}_0] + \mathrm{cum}[\widehat{H}_0^*, \widehat{H}_2, \widehat{H}_1^*, \widehat{H}_1] + \mathrm{cum}[\widehat{H}_0^*, \widehat{H}_2, \widehat{H}_2^*, \widehat{H}_2] + \mathrm{cum}[\widehat{H}_0^*, \widehat{H}_1, \widehat{H}_0^*, \widehat{H}_1] + \mathrm{cum}[\widehat{H}_1^*, \widehat{H}_2, \widehat{H}_1^*, \widehat{H}_2]\Big\} \quad (25)$$

6.2. Reduction of error of virtual correlation matrix 2

In the calculation of the fourth-order cumulants as the matrix elements of the virtual correlation matrix, the frequency characteristics are observed for the same time sequence, as in Eq. (15). In the calculation of the reflected wave component, however, the phase information is maintained the same, even if each of the two frequency characteristics is shifted by k [10]. Consequently, the time series are shifted m times, and the arithmetic mean of the results is used as the matrix element of the virtual correlation matrix:

$$C_2 = \frac{1}{m+1}\sum_{k=0}^{m} \mathrm{cum}[\widehat{H}^*(0, i-k), \widehat{H}(2, i-k), \widehat{H}^*(0, i), \widehat{H}(0, i)] \quad (26)$$

7. Performance Evaluation

7.1. Parameters of computer simulation

The performance of the proposed method was evaluated by computer simulation. Table 1 shows the parameters of the multiple delayed waves (signal waves) used as the object of estimation. The exponential type [11] is used as the delay profile. The Rice coefficient is the power ratio between the deterministic reflected signal component and the scattered signal component with stochastic fluctuation. For the observation noise, the signal power is summed up for all arriving waves, and six values are used, where the SNR (signal power-to-observed noise power ratio) is varied from 5.0 dB to 30.0 dB in 5.0-dB steps.

It is assumed that the mobile station (transmitting station) is moving at a speed of 40.0 km/h. The Doppler frequency shift is assumed for each transmitting direction from the mobile station. It is assumed that there is no direct wave to the base station, and no particular direction of motion with respect to the base station is considered.

In the estimation of the signal waves of Table 1, it is assumed that the frequency characteristics are observed at three points on the frequency axis by a straight-line array antenna with three antenna elements. Figure 3 shows the placement of the observation points, and Table 2 shows the sampling parameters. A 7 × 7 virtual correlation matrix is generated from the data. One thousand sampling data are used to calculate the fourth-order cumulants. Forty-eight time-series shifts are applied in the error reduction discussed in Section 6.2. The total number of sampled data is 1048 × 7 = 7336.

Table 1. Arriving waves

	1st wave	2nd wave	3rd wave	4th wave
Center frequency [GHz]		1.9		
Delay time [μs]	0.6	1.0	1.9	2.6
Delay time difference [μs]	0.0	0.4	0.9	0.7
Direction of arrival [degree]	−10.0	8.0	25.0	−30.0
Signal strength [dB]	0.0	−3.0	−3.7	−4.0
Rice coefficient		2.0		
Moving velocity [km/h]		40.0		

7.2. Result of estimation

In practice, the number p of delayed waves must be estimated. In this simulation, however, this process is omitted. The information that $p = 4$ is given beforehand, and the estimation is performed.

Figures 4 to 6 show the estimates of the delay time, the direction of arrival, and the signal strength, respectively. One hundred trials are made for each SNR value. In Fig. 6, the signal strength is normalized to the power of the first

Table 2. Sampling items

Observation point interval on frequency axis	[kHz]	300
Observation bandwidth on frequency axis	[kHz]	900
Element interval of array antenna	[cm]	7.9
Size of array antenna	[cm]	23.7
Number of sampling points on time axis	[samples]	1048
Sampling interval on time axis	[ms]	1.0

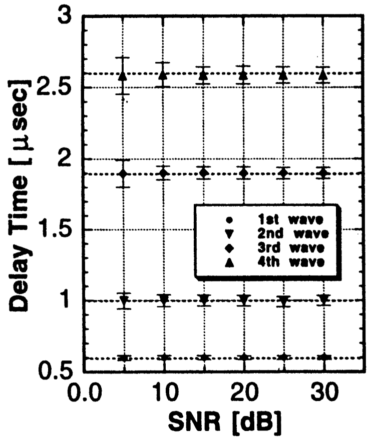

Fig. 4. Delay time estimation.

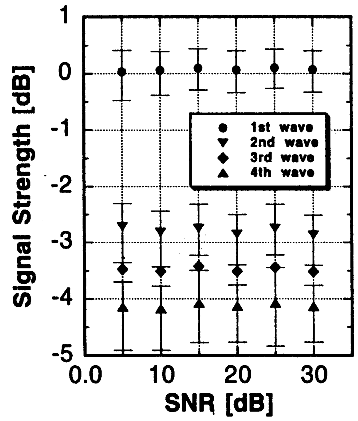

Fig. 6. Signal strength estimation.

wave. The standard deviation of the third wave is omitted since it overlaps with those of the second and fourth waves.

As to the estimation of the correspondence between the delay time and the direction of arrival, the success probability exceeds 95% for SNRs above 10 dB. The estimation is treated as a success when the estimated direction of arrival roughly agrees with the true direction of arrival. When the SNR is 5.0 dB (the power of the observation noise is 0.4 dB larger than the power of the first reflected component), there is a failure rate of 8%. It is seen from Fig. 4 that the first and second waves can be estimated with small variance and the results are close to the true values. The third and fourth waves, with small power, are estimated with large variance. The result, however, is unbiased. In other words, the average of a large number of estimated results (100 in this case) agrees with the true value.

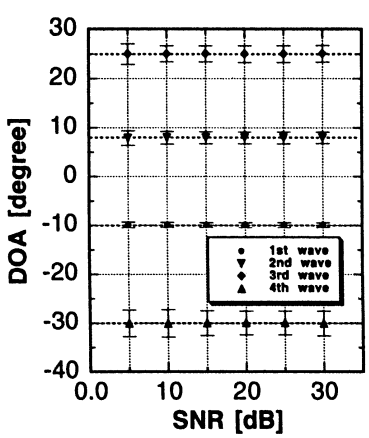

Fig. 5. DOA estimation.

The tendency is similar in the results of estimation of the direction of arrival shown in Fig. 5. The direction of arrival can be estimated for four waves, which is more than the number of antenna elements, i.e., three. When the number of observation points on the frequency axis is three, and the number of antenna elements in the straight-line array antenna is three in the estimation of the delay times and the directions of arrival, a maximum of two waves can be handled by the traditional MMP algorithm using the second-order moment. In other words, it is shown that the proposed method can estimate twice as many delayed waves.

In the estimation of the signal strength shown in Fig. 6, on the other hand, the variance of the estimated value is large, but it is seen that the power of the reflected components of the delayed waves can be roughly estimated. It should be noted that when the error reduction techniques described in sections 6.1 and 6.2 are not used, estimation of both the delay time and the direction of arrival is completely impossible.

8. Conclusions

The following method is proposed. The frequency characteristics of the multiple propagation path environment are observed by a straight-line array antenna. The observed data are processed by the method of fourth-order cumulants and the virtual correlation matrix is generated. By applying the MMP technique to the virtual correlation matrix, the delay time, the direction of arrival, the correspondence between the delay time and the direction of arrival, and the signal strength are estimated.

The proposed method uses the fourth-order cumulants. As a consequence, when the number of observation points on the frequency axis and the number of antenna elements of the straight-line array antenna are the same, the method can estimate at least twice as many delayed waves as in the case of the second-order moment. Next, a technique to reduce the error of the virtual correlation matrix is proposed. The performance is evaluated by a computer simulation, and the usefulness of the proposed method is demonstrated. In this paper, MMP is used as the superresolution technique, and the virtual correlation matrix based on the cumulants is applied. The virtual correlation matrix can in principle be introduced into other well-known superresolution methods such as MUSIC and ESPRIT. This point will be investigated in the future, together with the problem of estimating the number of delayed waves from the rank of the virtual correlation matrix.

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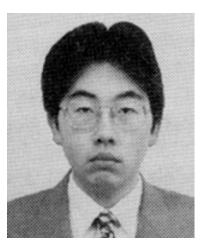
AUTHORS



Yuichi Shimamura (member). Education: Saitama University, Department of Electronic Engineering, B.E., 1995; doctoral program. Industry positions: NTT. Research interests: digital signal processing.

AUTHORS (continued) (from left to right)

Kiyomichi Araki (member). Education: Saitama University, Department of Electrical Engineering, B.E., 1971; Tokyo Institute of Technology, M.E., 1973, D. Eng., 1978. Academic appointments: Tokyo Institute of Technology, research associate, 1978; Saitama University, associate professor, 1985; University of Texas, visiting researcher, 1979–80; Saitama University, associate professor, 1985; University of Illinois visiting researcher, 1993–94; Tokyo Institute of Technology, Faculty of Engineering, professor, 1995. Research interests: information security, code theory, information theory, circuit theory, microwave circuits. Memberships: IEEE, Information Processing Society of Japan.

Jun-ichi Takada (member). Education: Tokyo Institute of Technology, Department of Electrical and Electronic Engineering, B.E., 1987, D. Eng., 1992. Academic appointments: Chiba University, Faculty of Engineering, research associate; Tokyo Institute of Technology, International Cooperative Center for Science and Technology, associate professor. Research interests: planar antennas, small antennas, mobile propagation; academic cooperation with Southeast Asia. Memberships: IEEE, ACES, SIAM. Recognitions: IEICEJ Paper Award, 1992; IEICEJ Young Engineer Award, 1993.