Sources of ToA Estimation Error in LoS Scenario

Marzieh Dashti†§, Mir Ghorashi‡, Katsuyuki Haneda§ Jun-ichi Takada‡

†Department of International Development Engineering, Tokyo Institute of Technology, Japan
§Aalto University School of Science and Technology, Finland

Email: {dashti, mir, takada}@ap.ide.titech.ac.jp
katsuyuki.haneda@tkk.fi

Abstract—Time-of-arrival (ToA) estimation technique used with ultra-wideband (UWB) transmission can be used for accurate indoor ranging. In the indoor multipath rich environment, these techniques often suffer from significant inaccuracy in ranging estimation. It is generally believed that the main cause of the ToA estimation error in the Line-of-sight (LoS) scenario is noise, usually modeled as white Gaussian noise. Through the established analytical framework for the analysis of the ToA estimation error, it is argued in this paper that the ToA estimation errors are caused by multipath interference and the fading statistics of the direct path. Therefore, variance of fading statistics of the direct path are considered on top of the noise variance for better prediction of the ToA estimation error. A comparison of predicted error with those obtained based on experimental measurements in an indoor environment confirms the validity of our analytical approach.

I. INTRODUCTION

Several publications discuss the topic of the Time-of-arrival (ToA) based ranging error statistics for ultrawideband (UWB). Gaussian model for ranging error in the line-of-sight (LoS) case and a mixture of exponential and Gaussian in non-line-of-sight (NLoS) case are reported in [1]. The bandwidth effects are studied in details in [2]. A data fitting is applied to a set of range errors in [3] resulting a polynomial representation of the distance dependent mean and variance, the range error is modeled as a normally distributed random variable (RV) for both LoS and NLoS. The common disadvantage of these models is that their approach is to conduct a measurement campaign, apply a ToA algorithm and perform data fitting on the obtained measurements. Hence, these models are environment dependent. Moreover, no reason was presented for the reported Gaussian models [1]-[3].

To assess the ranging error behavior over experimental data, a measurement campaign was conducted in an office room. It is shown in this paper that the Gaussian distribution model does not fit to the measured ranging error. It is due to the fact that Gaussian noise is not the only source of error in LoS scenario though it is generally assumed.

In this work an analytical approach is taken to analyze the ranging error similar to the method in [4]-[5]. Through the analytical analysis of the range estimation error, it is argued in this paper that the ranging errors in LoS links can be better predicted by multipath interference than by Gaussian noise.

We begin the paper with a brief review of the UWB ToA estimation algorithm in section II. Measurement scenario and experimental measured ranging error results are reported in section III. The analytical derivation of the range error is presented in section IV. The comparison of the analytical results with experimental measurement results also is given in this section. And finally section V summarize this paper.

II. TOA ESTIMATION ALGORITHM

The received ranging signal is obtained by superposition of the LoS and I multipath signals and is characterized as

\[ r(t) = \sum_{i=1}^{I} \sqrt{E_{tx}} \alpha_i p_{tx}(t - \tau_i) + n(t) \]

where \( \alpha_i \) and \( \tau_i \) are complex path gain and propagation delay of the \( i \)'th path. The Tx sends out a ranging signal \( \sqrt{E_{tx}} p_{tx}(t) \) where \( p_{tx} \) is the transmitting pulse waveform with normalized energy and \( E_{tx} \) is the pulse energy. Depending on the phase of multipath constructive or destructive interference can occur. Moreover, \( n(t) \) is zero mean Gaussian random noise with variance \( \sigma_n^2 = N_0/2 \). \( \tau_i = l_i/c \) where \( c \) is the velocity of light and \( l_i \) is the \( i \)'th path length. Let the expected arrival time of the first arrival path (FAP) be \( \tau_{FAP} = l_0/c \) where \( l_0 \) is the LoS path length.

The energy detector (ED) receiver is considered throughout the paper. The ED output samples can be expressed
as
\[ z(n) = \int_{nT}^{(n+1)T} |r(t)|^2 dt \]  
(2)
where \( T \) is sampling period, \( n \in 1, \ldots, N \) is sample indices and \( N \) is the total number of received samples. The integrator performs the sampling operation by successively integrating the squared received signal.

A simple technique to detect the FAP is to compare the output of the ED, \( z(n) \), with a threshold whose value has to be optimized according to the environments and the Tx-Rx distances. In the applied ToA estimation algorithm, the first threshold-exceeding sample index can be regarded as detected FAP sample index and the arrival time of this sample, \( \tau_D \), is corresponded as the ToA estimate:
\[ \tau_D = n_D T, \]
\[ n_D = \arg \min_n (z(n) > \xi), \]  
(3)
where \( n_D \) is the sample index of detected FAP and \( \xi \) is a threshold.

### III. MEASURED RANGING ERROR

To assess the ranging behavior over experimental data, a measurement campaign was conducted in an office room. Rx antenna was fixed at the corner of the room, whereas the Tx antenna could be positioned at almost any place in the room by the aid of a precise scanner covering the whole areas of the room. Tx-Rx distances varied from 0.6 to 9.3 m. In total, 4200 spatial samples of transfer function were measured on the Tx side. The channel transfer function of a single Tx-Rx combination was measured by a vector network analyzer (VNA). Measurement specifications are summarized in [7]. The maximum detectable ToA was 200 ns, and was sufficient to capture dominant propagation paths. All the analysis in this study were based on allocated sub-bands in the IEEE 802.15.4a standard, specified in Table 39i-UWB PHY in [8]. For the sake of conciseness only the results obtained from channel 9 of the standard \( f_c = 7.9 \) GHz and \( B = 0.5 \) GHz is reported in this paper.

ToA estimation algorithm is applied to all measured received signals. Assuming that Tx and Rx nodes are positioned at known coordinates and are perfectly synchronized, the measured distance between the Tx and the Rx nodes can be obtained from:
\[ d_m = \tau_D \times c, \]  
(4)
The ranging error \( e \) was then obtained from:
\[ e = d_m - d, \]  
(5)
where \( d \) is the real distance between Tx and Rx antennas obtained from known coordinates of Tx and Rx node.

Fig. 1 shows the example of a measured received signal where the ToA of FAP is estimated wrongly from expected ToA. Fig. 2 shows the CDF of difference between estimated ToA and strongest path delay from expected ToA. Fig. 1 and 2 also depicted that due to the effect of multipath interference the strongest path (SP) is not necessarily the FAP even under the LoS condition. Multipath interference leads to fading and causes the SP spread over the delay axis.

#### A. Measurement-based error model

In previous reported works, the range error is modeled as a Gaussian model in the LoS case [1]-[3]. Similar to the approach introduced in [1] for modeling the ranging error, we assume the Gaussian distribution with zero mean and the variance derived from measurement results. Fig. 4 compares the CDF of measured ranging error and
Gaussian model. It shows that assuming normalized error with Gaussian distribution is not a good assumption.

IV. ANALYSIS OF RANGING ERROR

In this section, the sources of ranging error through analysis is argued. Closed form expressions of the ToA estimation error are presented using similar approach to [4]. A new insight added in the present analyses is that the multipath propagation is considered as source of ToA estimation error on the top of the noise. For analytically deriving the error, first the probability of detection of a certain sample is derived. The method begins the search at \( n = 1 \). Probability of detecting a certain sample as an FAP, \( n_{D} \), is equal to the probability of that sample being detected multiplied by the probability of none of the previous samples is detected. It is therefore formulated as

\[
P_D(n_D) = \left( \prod_{n=1}^{n_D-1} p(z(n) < \xi) \right) \times p(z(n_D) > \xi)
\]

(6)

First term in the right side of Eq. (6) is the probability of non of the samples earlier than \( n_{D} \) being detected and second term is the probability of \( n_{D} \) being detected. In Eq. 6, \( n_{D} = n_{FAP} \) corresponds to correct detection. Samples arrived earlier than \( n_{FAP} \), i.e. \( n \in 1, 2, ..., n_{FAP} - 1 \), are noise-only sample; where \( z(n) \) is a RV which has a centralized Chi-square distribution with two degrees of freedom. FAP sample and samples arriving after the FAP are signal-plus-noise samples where \( z(n) \) has a non-centralized Chi-square distribution with two degrees of freedom. Let \( n \in n_{FAP}, n_{FAP} + 1, ..., n_{FAP} + N_e \) denote indices of signal-plus-noise samples, where \( N_e \) is the number of samples containing a significant amount of energy. On the other hand samples with indices \( n \in n_{FAP} + N_e + 1, ..., N \) are in the tail of the channel impulse response which is likely to contain only noise. The CDF of these centralized Chi-square, \( \chi \), and non-centralized Chi-square, \( \chi' \), RVs are given by [9], [10].

\[
\chi(\xi) = p(z(n) < \xi) = 1 - \exp\left( -\frac{\xi}{2\sigma_n^2} \right)
\]

(7)

\[
\chi'(\alpha_n, \xi) = p(z(n) < \xi) = 1 - Q_1 \left( \frac{\sqrt{\alpha_n}}{\sigma_n}, \frac{\sqrt{\xi}}{\sigma_n} \right)
\]

(8)

where \( Q_1(.) \) denotes the Marcum-Q function with parameter \( x \), and \( \alpha_n \) is the gain of the \( n \)’th sample, whose PDF varies with \( n \).

A numerical approach is applied for calculation of (6). If \( n_{D} < n_{FAP} \), we have,

\[
P_D(n_D) = (\chi(\xi))^{n_{D}-1} (1 - \chi(\xi))
\]

(9)

while on the other hand if \( n_{D} = n_{FAP} \),

\[
P_D(n_{FAP}) = \chi(\xi)^{n_{FAP}-1} \times \int_{\alpha_{n_{FAP}}} (1 - \chi'(\alpha_{n_{FAP}}, \xi)) p(\alpha_{n_{FAP}}) d\alpha_{n_{FAP}}
\]

(10)

If \( n_{D} > n_{FAP} \) we can further consider two conditions. If \( n_{D} - n_{FAP} \leq N_e \),

\[
P_D(n_{D}) = (\chi(\xi))^{n_{D}-n_{FAP}-1} \times \left( \prod_{n=n_{FAP}}^{n_{D}-1} (\chi(\alpha_n, \xi)) p(\alpha_n) d\alpha_n \right) \times \int_{\alpha_{n_{D}}} (1 - \chi'(\alpha_{n_{D}}, \xi)) p(\alpha_{n_{D}}) d\alpha_{n_{D}}
\]

(11)

while if \( n_{D} - n_{FAP} > N_e \),

\[
P_D(n_{D}) = (\chi(\xi))^{n_{D}-n_{FAP}-1} \times \left( \prod_{n=n_{FAP}}^{n_{D}-N_e-1} (\chi(\alpha_n, \xi)) p(\alpha_n) d\alpha_n \right) \times \int_{\alpha_{n_{FAP}+N_e}} \chi'(\alpha_n, \xi) p(\alpha_n) d\alpha_n
\]

(12)

The probability of no-detection, i.e. none of the samples is detected, is derived as Eq. 13. No-detection probability is practically close to zero.

\[
P_{ND} = 1 - \sum_{n=1}^{N} P_D(n)
\]

(13)

Ranging error can be associated to each sample as

\[
e(n) = P_D(n) \left( |n - n_{FAP}| c T \right).
\]

(14)

In Eq. 14, the error corresponding to each sample is weighted by the probability of detecting that particular sample.

In this paper in order to carry out the analytical evaluation of the detection probabilities, the gain PDFs, \( p(\alpha_n) \), are obtained via ray-tracing simulations. A 3D ray-tracing simulation is accomplished for an indoor environment with dimensions 10 \( \times \) 10 \( \times \) 3 m\(^3\) for 4200 positions of Rx uniformly distributed on the plane of 1.3 m height where Tx is assumed with the same height. For the sake of a comparison of simulated and measurement, the simulation setting is similar to our established measurement scenario. Only LoS path and six first-order reflected paths from ceiling, floor and walls are considered.

Fig. 3 shows PDF of sample gains in four different excess delay with respect to FAP, i.e. for \( n - n_{FAP} = 0, 5, 10, 15 \), obtained from the ray-tracing simulation.

A. SOURCES OF RANGING ERROR

In the analysis first we consider noise as the source of ToA estimation error. Hence noise variance, \( \sigma_n \), is
used in equations 7-8. However in the indoor multipath rich environment, the variance of the signal due to multipath spatial fading is much larger than $\sigma_n$. The FAP gain has the variance of $-79$ dB which is higher than $\sigma_n = -100$ dB. Hence, not only the presence of the noise but also fading statistics of the FAP should be considered in the ToA estimation error analysis. For further analysis, we consider the Rician fading with a moderate $K$-factor value and variance $\sigma_F$. Fig. 4 compared the range error from measurements and from analytical expressions considering the noise variance, $\sigma_n = -100$ dB, and the FAP path gain variance, $\sigma_F = \sigma_{FAP} = -79$ dB $\gg \sigma_n$. The error obtained from analytical error model considering $\sigma_F$ fitted closely to those from experiment. It confirmed our conclusion that the fading variance, $\sigma_F$, for error analysis should be considered on top of the noise variance, $\sigma_n$.

V. CONCLUSION

In this work an analytical approach is taken to analyze the ranging error behaviour. A new insight added in the present analyses is that the multipath are considered as source of ToA estimation error on the top of the noise. It is shown that not only the presence of the noise but also fading statistics of the FAP should be considered in the ToA estimation error analysis. A comparison with results based on experimental measurements in an indoor environment confirms the validity of our analytical approach.

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