Modified Array Calibration for Precise Angle-of-Arrival Estimation

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Abstract

In this paper, an evaluation of calibration data for the antenna array based on measurement data is proposed. A simple calibration method to compensate amplitude and phase errors between array elements is presented. A set of measurement data is collected from a single source whose angle of arrival (AOA) is varied in a known direction. To demonstrate the effectiveness of the calibration method, AOA is estimated based on the MUSIC algorithm. However, from the AOA estimation result, it is found that the calibration data are effective only to estimate AOA close to the angle of the signal used to calculate the calibration data. Moreover, we found that the phase differences of calibration data change with AOA while the amplitude differences do not. Therefore, we propose the least square problem to approximate the phase differences as a function of AOA and then modify the calibration data. The result illustrates the improvement of the calibration data when the least square function is applied to the phase differences.

Keywords: Antenna array, Array calibration, AOA estimation

1 Introduction

Angle of arrival (AOA) information of an impinging signal on an antenna array can be utilized for many applications such as smart antenna and mobile localization system. Especially, for mobile localization, a precise AOA estimate is required. For example, to localize a mobile terminal within a 10-m accuracy when the distance between the antenna array and a mobile terminal is 4 km, the AOA estimation error less than 0.15 degrees is needed. In the last few decades, a variety of high-resolution algorithms for AOA estimation have been extensively discussed in the literature [1]-[3]. Nevertheless, these algorithms have superior resolution over conventional AOA estimation techniques, e.g. [4], only when computer simulations or a comparatively limited number of experiment data processing systems are used to estimate AOA. One of the main reasons of limitation to use these high-resolution algorithms in real applications is that the difficulties to calibrate the data collection systems.

In this paper, the calibration method of the antenna array based on the measurement data is proposed. To compensate the amplitude and phase errors between the array elements, the amplitude and phase responses of each element are needed to be known or correctly estimated. One simple way to estimate the amplitude and phase response is to place the transmitter at a known location whose angle of arrival is known at the antenna array, then comparing these amplitude and phase responses to the ideal ones. Then, the calibration data can be obtained from these amplitude and phase differences. To evaluate the effectiveness of the calibration method, AOA is estimated based on Multiple Signal Classification (MUSIC) algorithm [1]. Herein, the calibration data is considered to be effective when the error of AOA estimation is less than approximately 0.15 degrees. Nevertheless, we found that the calibration data are effective only to estimate AOA which is near the AOA used to make the calibration data. Additionally, we found that the phase differences of the calibration data change with AOA while the amplitude differences do not change. Therefore, we propose the least square problem to fit the phase differences as a function of AOA and then modify the calibration data. The result of AOA estimation shows the improvement of this modified calibration method.

This paper is organized as follows. In Section 2, the signal model is explained followed by the experiment setup, in Section 3. Next, the modified calibra-
ation method is described in Section 4. Then, in Section 5, the evaluation of the array calibration method is investigated and finally, the conclusion is presented in Section 6.

2 Signal Model and MUSIC Algorithm

2.1 Signal Model

Consider $K$-narrowband signal sources impinging at an $M$-element uniform linear array (ULA), the received signal vector can be modeled as [5]

$$ x(t) = CA\mathbf{s}(t) + \mathbf{n}(t), \quad (1) $$

where $\mathbf{s}(t)$ is the $K \times 1$ signal waveform vector, $\mathbf{n}(t)$ is the $M \times 1$ noise vector and $\mathbf{A}$ is the $M \times K$ steering matrix; $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_K)]$ where $\mathbf{a}(\theta)$ denotes a steering vector defined by

$$ \mathbf{a}(\theta) = [1, ..., \exp(-j2\pi d(m-1)\sin(\theta)/\lambda)]^T. \quad (2) $$

$d$ is the inter-element spacing and $\lambda$ is the signal carrier wavelength. $\mathbf{C}$ is the $M \times M$ calibration matrix constructed for removing the array imperfection error, e.g., the amplitude and phase mismatch of array elements and the mutual coupling caused by electromagnetic coupling between array elements. However, we assume that no mutual coupling effect in this paper. The calibration matrix can be simply constructed from the differences between the measured amplitude and phase of each element with the ideal ones. For instance, when the signal impinging at the antenna array of $0$ degrees is collected, the signal amplitude and phase of each element are theoretically the same and considered as the ideal amplitude and phase of elements. Therefore, the calibration matrix can be obtained by the amplitude and phase differences between the measurement and the ideal case. In this case, the calibration matrix can be written as

$$ \mathbf{C} = \text{diag}[\frac{\alpha_1}{\sigma}, ..., \frac{\alpha_1}{\sigma}, ..., \frac{\alpha_K}{\sigma}, ..., \frac{\alpha_K}{\sigma}], \quad (3) $$

where $\alpha_1, ..., \alpha_M$ and $\phi_1, ..., \phi_M$ refer to as the signal amplitude and phase associated at each array element and the first element is considered as the reference.

In terms of the signal model in equation (1), the output covariance matrix can be expressed as [6]

$$ \mathbf{R} = E[x(t)x^H(t)] = \tilde{\mathbf{A}}\mathbf{P}\tilde{\mathbf{A}}^H + \sigma^2\mathbf{I} = \mathbf{U}\mathbf{A}\mathbf{U}^H. \quad (4) $$

where $\tilde{\mathbf{A}} = \mathbf{CA}$ and $E[\cdot]$ represents statistical expectation,

$$ \mathbf{P} = E[s(t)s^H(t)] \quad (5) $$

is the source covariance matrix and

$$ \sigma^2\mathbf{I} = E[n(t)n^H(t)] \quad (6) $$

is the noise covariance matrix that reflects the uncorrelated noise among all the elements and having a common variance at all elements.

In practice, it is clear that only the estimated covariance matrix is available and can be expressed as

$$ \hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^{N} x(n)x^H(n), \quad (7) $$

where the sample data $x(n)$ are observed over $N$ snapshots of the actual antenna array output; $t = 1, 2, ..., N$.

An eigendecomposition of $\hat{\mathbf{R}}$ can be written as

$$ \hat{\mathbf{R}} = \hat{\mathbf{U}}\hat{\mathbf{A}}\hat{\mathbf{U}}^H + \hat{\mathbf{U}}\hat{\mathbf{n}}\hat{\mathbf{U}}^H, \quad (8) $$

where $\hat{\mathbf{U}}_s$ is the matrix which contains signal eigenvectors shown as

$$ \hat{\mathbf{U}}_s = [\hat{\mathbf{u}}_1, ..., \hat{\mathbf{u}}_K], \quad (9) $$

$\hat{\mathbf{U}}_n$ is the matrix which contains noise eigenvectors shown as

$$ \hat{\mathbf{U}}_n = [\hat{\mathbf{u}}_{K+1}, ..., \hat{\mathbf{U}}_M], \quad (10) $$

and $\hat{\mathbf{A}}_s$ and $\hat{\mathbf{A}}_n$ are diagonal matrices of real eigenvalues ordered such that $\lambda_1 \geq \lambda_2, ..., \geq \lambda_K > \sigma^2$ and $\lambda_{K+1} = ... = \lambda_M = \sigma^2$, respectively.

2.2 MUSIC Algorithm

Since the steering vectors corresponding to signal components are orthogonal to the noise subspace eigenvectors;

$$ \hat{\mathbf{U}}_n^H\mathbf{a}(\theta) = 0, \quad (11) $$

where $\theta$ corresponding to AOs of the impinging signal: $\theta \in \{\theta_1, ..., \theta_K\}$, AOAs can be estimated by locating the peaks of the MUSIC spatial spectrum given by

$$ P_{\text{MUSIC}}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{a}(\theta)}. \quad (12) $$

3 Experiment Setup

The data were obtained from a field experiment at Hokkaido, Japan as shown in figure 1. Two parallel uniform linear arrays were mounted on the top of a 10-m-high building. The receiver antenna is illustrated in figure 2 and its parameters are shown in Table 1. The transmitter had a 1-m-high single element antenna place at the known position in an open area with LOS condition. The transmitter antenna and its parameters are illustrated in figure 3 and Table 2, respectively. The carrier frequency is 1.74 GHz. The data were collected at the receiver antenna array, then transferred to an analog to digital converter.
(ADC), FPGA and finally processed by a digital signal processor (DSP). The block diagram of the receiver experiment system is shown in figure 4. Although AOAs can be processed and estimated by the DSP equipped with a given software, the output data from FPGA can also be used to estimate the signal parameters with other algorithms. In this paper, we use the latter case to estimate AOAs with the MUSIC algorithm. The experiments were repeated 17 times by changing the position of the transmitter whose known angles of arrival relative to the receive antenna array were $-6, -5, -2, -0.5, -0.2, -0.1, -0.05, 0, 0.05, 0.1, 0.2, 0.5, 1, 2, 5$ and $6$ degrees and denoted as $\theta_1, \theta_2, \ldots, \theta_N$, respectively.

![Figure 1: Field of Experiment](image1)

![Figure 2: Receiver Antenna Array](image2)

<table>
<thead>
<tr>
<th>Table 1: Specifications of the receive antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Elements</td>
</tr>
<tr>
<td>Spacing of Element</td>
</tr>
<tr>
<td>Antenna Elements</td>
</tr>
<tr>
<td>Antenna Gain</td>
</tr>
</tbody>
</table>

4 Modified calibration data based on measured data

Nevertheless, we found that the above calibration data are effective only to estimate AOA near the angle of the signal used to calculate the calibration data as shown in figure 5. Additionally, the phase differences of the calibration data change with AOA while the amplitude differences do not change with AOA. Therefore, we propose the linear least square (LS) approach to approximate the phase differences as a function of AOA and modify the calibration data. Here, the linear LS fitting to the phase difference is presented. First, we derive the phase difference of each element from measured signal whose arriving angles are known as $\theta_l, l = 1, \ldots, L$. Next, the phase difference of each element is approximated by the linear LS fitting method. In this paper, the first-degree, the second-degree and the third-degree polynomials are used to approximate the best fit to the measured phase difference.

![Figure 3: Transmitter Antenna](image3)

<table>
<thead>
<tr>
<th>Table 2: Specifications of the transmitter antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Elements</td>
</tr>
<tr>
<td>Antenna Gain</td>
</tr>
<tr>
<td>Polarization</td>
</tr>
</tbody>
</table>

4.1 LS data fitting by the first-degree polynomial

Fitting the data by the first-degree polynomial or a straight line is the most common data fitting problem. In our case, the phase difference of each element approximated by this problem can be written as

$$\phi_{m,1}\theta = \tilde{a}_{1m} + \tilde{a}_{0m}.$$  \hspace{1cm} (13)

$\tilde{a}_{1m}$ and $\tilde{a}_{0m}$ are $m$-th element unknown parameters approximated by [7],

$$\begin{bmatrix} \tilde{a}_{1m} \\ \tilde{a}_{0m} \end{bmatrix} = (\Theta_{1st}^T\Theta_{1st})^{-1}\Theta_{1st}^T\Phi_m,$$  \hspace{1cm} (14)

where $\Theta_{1st}$ and $\Phi_m$ are respectively defined by

$$\Theta_{1st} = \begin{bmatrix} \theta_1 & 1 \\ \theta_2 & 1 \\ \vdots & \vdots \\ \theta_L & 1 \end{bmatrix},$$  \hspace{1cm} (15)
and

\[ \Phi_m = \begin{bmatrix} \phi_{m,1} \\ \phi_{m,2} \\ \vdots \\ \phi_{m,L} \end{bmatrix}. \]  

(16)

\( \phi_{m,1st}(\theta) \) is the phase difference of the \( m \)-th element associated with the arriving angle of \( \theta_l \); \( l = 1, ..., L \) when the first-degree polynomial is applied. Finally, the modified calibration data as a function of AOA can be determined by

\[ C(\theta) = \text{diag}[C_1(\theta), ..., C_M(\theta)], \]  

(17)

where \( C_m(\theta) = \alpha_m e^{j\phi_{m,1st}(\theta)} \) is the calibration data of the \( m \)-th element in which \( \alpha_m \) is the \( m \)-th element amplitude of the calibration data when the signal arrives at the antenna array at 0 degrees and \( m = 1, ..., M \).

![Figure 4: Block diagram of the experiment system in the receiver side](image)

Figure 4: Block diagram of the experiment system in the receiver side

![Figure 5: Comparison of AOA estimation error when applying the calibration data constructed by signals whose AOA is -5, -2, 0, 2 and 5 degrees, respectively.](image)

Figure 5: Comparison of AOA estimation error when applying the calibration data constructed by signals whose AOA is -5, -2, 0, 2 and 5 degrees, respectively.

### 4.2 LS data fitting by the second-degree polynomial

To approximate the phase difference by the second-degree polynomial, the phase difference of each element can be modeled as

\[ \phi_{m,2nd}(\theta) = \tilde{b}_{2m}\theta^2 + \tilde{b}_{1m}\theta + \tilde{b}_{0m}. \]  

(18)

\( \tilde{b}_{2m}, \tilde{b}_{1m} \) and \( \tilde{b}_{0m} \) are three unknown parameters of \( m \)-th element which can be approximated by,

\[ \begin{bmatrix} \tilde{b}_{2m} \\ \tilde{b}_{1m} \\ \tilde{b}_{0m} \end{bmatrix} = (\Theta_{2nd}^T\Theta_{2nd})^{-1}\Theta_{2nd}^T\Phi_m, \]  

(19)

where \( \Phi_m \) is as same as equation 16 while \( \Theta_{2nd} \) can be defined by

\[ \Theta_{2nd} = \begin{bmatrix} \theta_1^2 & \theta_1 & 1 \\ \theta_2^2 & \theta_2 & 1 \\ \vdots & \vdots & \vdots \\ \theta_L^2 & \theta_L & 1 \end{bmatrix}. \]  

(20)

\( \phi_{m,2nd}(\theta) \) is the phase difference of the \( m \)-th element associated with the arriving angle of \( \theta_l \); \( l = 1, ..., L \) when the second-degree polynomial is applied. Then, the modified calibration data can be obtained by substituting \( C_m(\theta) = \alpha_m e^{j\phi_{m,2nd}(\theta)} \) in equation 17.

### 4.3 LS data fitting by the third-degree polynomial

In the same manner, the phase difference of each elements approximated by the third-degree polynomial can be written as

\[ \phi_{m,3rd}(\theta) = \tilde{c}_{3m}\theta^3 + \tilde{c}_{2m}\theta^2 + \tilde{c}_{1m}\theta + \tilde{c}_{0m}. \]  

(21)

\( \tilde{c}_{3m}, \tilde{c}_{2m}, \tilde{c}_{1m} \) and \( \tilde{c}_{0m} \) are four unknown parameters of \( m \)-th element which can be approximated by,

\[ \begin{bmatrix} \tilde{c}_{3m} \\ \tilde{c}_{2m} \\ \tilde{c}_{1m} \\ \tilde{c}_{0m} \end{bmatrix} = (\Theta_{3rd}^T\Theta_{3rd})^{-1}\Theta_{3rd}^T\Phi_m, \]  

(22)

where \( \Phi_m \) is as same as equation 16 while \( \Theta_{3rd} \) can be defined by

\[ \Theta_{3rd} = \begin{bmatrix} \theta_1^3 & \theta_1^2 & \theta_1 & 1 \\ \theta_2^3 & \theta_2^2 & \theta_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \theta_L^3 & \theta_L^2 & \theta_L & 1 \end{bmatrix}. \]  

(23)

\( \phi_{m,3rd}(\theta) \) is the phase difference of the \( m \)-th element associated with the arriving angle of \( \theta_l \); \( l = 1, ..., L \) when the third-degree polynomial is applied. Then, the modified calibration data can be obtained by substituting \( C_m(\theta) = \alpha_m e^{j\phi_{m,3rd}(\theta)} \) in equation 17.
5 Evaluation of array calibration

To evaluate the effectiveness of the calibration method, AOA is estimated based on the MUSIC algorithm. We found that the higher the order LS fitting is used to approximate the phase differences, the better improvement is obtained as shown in figure 6.

![Figure 6: Comparison of AOA estimation error when applying LS to the calibration data](image)

6 Conclusion

It is found that the LS approximation of phase differences to the calibration data seems to be effective for the AOA estimation. However, estimation errors in some measurement AOAs are still high which might be due to imperfect calibration data affected, for instance, by electromagnetics diffraction or instability of the antenna array. To correct the calibration error, the properties of the antenna array are needed to be clarified for further works.

References


