# Signal Detection for Analog and Digital TV Signals for Cognitive Radio

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**Abstract** In IEEE 802.22 wireless RAN system, the signal detection mechanism is required for identifying occupied and vacant TV channels. In this paper, we consider the signal detection based on the Neyman-Person approach. With several channel and signal models, the probability of detection and false alarm are studied as functions of signal-to-noise ratio (SNR).

**Key words** Cognitive Radio, IEEE 802.22, Spectrum Detection

## 1. Introduction

Recently, a Cognitive Radio (CR) has been proposed to improve the current static spectrum access technique by allowing an unlicensed system to share the same radio spectrum resources with the primary system at the same time and the same location.

The operating characteristics of CR can be altered based on interaction with the environment in which it operates [1]. The CR is seen as a smart radio that can detect the presence or absence of the primary signal and use only an unoccupied channel of the primary system in order to avoid interference with the primary users.

The first application of CR is studied by the IEEE 802.22 Working Group (WG). In November 2004, the IEEE 802.22 WG have proposed to standardize a fixed wireless access system which is based on the CR technology [2], [3]. It is called an IEEE 802.22 Wireless Regional Area Network (WRAN). The IEEE 802.22 WRAN aims at providing the wireless broadband access to rural areas, as well as to urban areas where the spectrum usage is low.

The IEEE 802.22 WRAN will share the same spectrum resources with the legacy TV system.

Regarding of application of IEEE 802.22 WRAN, the detection and sensing of radio spectrum is the critical issue.

The key challenge of CR for the IEEE 802.22 WRAN is the detection of weak TV signals in noise with a very small probability of miss detection.

The goal of this paper is to provide a comprehensive study of Neyman-Pearson detection which is used to detect a known deterministic signal in white Gaussian noise. It can maximize the probability of detection for a given probability of false alarm. We will apply this approach to detect the digital TV signal at very low signal-to-noise ratio.

We organize the rest of this paper as follows. In Section 2, we review the Neyman-Pearson detection. The detection of TV signal is explained in Section 3. Simulation results and future work are presented in Section 4 and Section 5, respectively. Finally, our conclusion is presented in Section 6.

## 2. Neyman-Pearson Detection

### 2.1 Background

We suppose a known deterministic signal $s[n]$ is corrupted by the white Gaussian noise $w[n]$ with zero mean and variance $\sigma_w^2$ [4].

In general, we consider the received signal $x[n]$ of the following two forms

$$H_0 : x[n] = w[n], \qquad \text{signal is absent} \qquad (1)$$

$$H_1 : x[n] = s[n] + w[n], \qquad \text{signal is present} \qquad (2)$$

where $n = 0, 1, 2, ..., N-1$, $N$ is the sample index and $s[n]$ is the primary signal that is required to detect.

In the absence of coherent detection, the signal samples $s[n]$ can be also model as a Gaussian random process with zero mean and variance $\sigma_s^2$ [5].

The decision statistic of Neyman-Pearson is to determine between the hypotheses $H_0$ and $H_1$ based on the observations of $X = [x[0]\ x[1]\ ...\ x[N-1]]^T$.

The Neyman-Pearson detection approach states that the signal is present if the likelihood ratio exceed the threshold, i.e.

$$L(X) = \frac{p(X|H_1)}{p(X|H_0)} > \gamma \qquad (3)$$

where $p(X|H_1)$ and $p(X|H_0)$ are the probability density function of hypotheses $H_1$ and $H_0$ respectively, and $\gamma$ is the threshold. Note that $\gamma$ can be computed to satisfy the probability of false alarm.

After mathematical simplification of equation (3), finally, the decision of Neyman-Pearson approach is given by:

$$T = \mathrm{Re}(\sum_{n=0}^{N-1} x[n]s^*[n]) > \gamma', n = 0, 1, ...N-1 \tag{4}$$

### 2. 2 Evaluation

Equation (4) shows that if the distribution $T > \gamma'$, the signal is present and if the distribution $T < \gamma'$, the signal is absent.

Generally, the Neyman-Pearson detection approach has two errors. The first error is happened when the primary signal is absent but the detector declares that the signal is present. This is called a probability of false alarm ($P_{\mathrm{FA}}$).

The second error is appeared when the channel is occupied, but the detector declares that the channel is vacant. This is called a probability of miss detection ($P_{\mathrm{MD}}$).

However, it is well known that under the Neyman-Pearson criteria, the performance of detection is measured by a resulting pair of probability of detection and the probability of false alarm. Each pair is associated with the particular threshold $\gamma$ that tests the decision value $T$.

Under either hypotheses, $x[n]$ is Gaussian and since $T$ is a linear combination of Gaussian random variables, $T$ is also a Gaussian. Therefore, we can approximate distribution $T$ as a Gaussian distribution with mean and variance
$T \sim \mathcal{N}(\epsilon_0, \sigma_0^2)$ under $H_0$ and $T \sim \mathcal{N}(\epsilon_1, \sigma_1^2)$ under $H_1$. Note that the mean and variance of value $T$ will be described in Section 3. Finally, the $P_{\mathrm{FA}}$ and the probability of of detection ($P_{\mathrm{D}}$) can be computed as,

$$P_{\mathrm{FA}} = Q(\frac{\gamma - \epsilon_0}{\sqrt{\sigma_0^2}}) \tag{5}$$

$$P_{\mathrm{D}} = Q(\frac{\gamma - \epsilon_1}{\sqrt{\sigma_1^2}}) \tag{6}$$

For the fixed number of sample index $N$, the $P_{\mathrm{D}}$ can be evaluated by substituting the threshold in Eq. (6). Each threshold corresponds to a pair ($P_{\mathrm{FA}}$,$P_{\mathrm{D}}$), representing the receiver operating characteristics (ROC).

## 3. Detection of TV signal

We suppose that a weak digital TV signal is corrupted by white Gaussian noise. The goal of our work is to determine which TV channels are occupied and which are vacant. In order to achieve this goal, we are considering on two methods; the energy detector and the replica-correlator.

### 3. 1 Energy Detector

Energy detector can be performed in both time domain and frequency domain. The NP detector computes the energy in the received data and compares it to the threshold. Intuitively, if the signals is present, the energy of the received data increases. A decision value of the energy detector is

$$T = \sum_{n=0}^{N-1} |x[n]|^2 > \gamma, n = 0, 1, ...N-1 \tag{7}$$

The energy detector is depicted in the Figure 1. To measure the signal energy in a particular frequency region in time domain, the received signal is applied to a bandpass filter and then convert to numerical signal which is then measured.

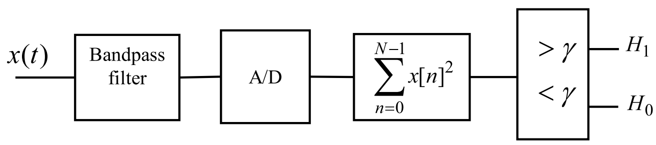


Figure 1 Energy detector

For simplicity of computation, we assume that the noise sample $w[n]$ and the signal sample $s[n]$ are independent. Since the received sample $x[n]$ is a linear combination of $w[n]$ and $s[n]$, therefore $x[n]$ is also independent. However, the $x^2[n]$ is a sequence of independent and identically distribute random variables. After some mathematical steps, the mean and variance of $x^2[n]$ can be expressed as the following

$$\mathrm{E}[x^2[n]|H_0] = \sigma_w^2 \tag{8}$$

$$\mathrm{var}[x^2[n]|H_0] = 2\sigma_w^4 \tag{9}$$

$$\mathrm{E}[x^2[n]|H_1] = (\sigma_w^2 + \sigma_s^2) \tag{10}$$

$$\mathrm{var}[x^2[n]|H_1] = 2(\sigma_w^2 + \sigma_s^2)^2 \tag{11}$$

Since the detector must be able to detect the TV signal at very low SNR, the number of required sample index should be large. If the sample index $N$ is large, we can apply the central limit theorem to the decision value of the energy detector $T$, and then it can be approximated as a Gaussian random variable with mean and variance

$T \sim \mathcal{N}(N\sigma_w^2, 2N\sigma_w^4)$ under $H_0$

$T \sim \mathcal{N}(N(\sigma_w^2 + \sigma_s^2), 2N(\sigma_w^2 + \sigma_s^2)^2)$ under $H_1$

Finally, the $P_{\mathrm{FA}}$ and $P_{\mathrm{D}}$ can be evaluated as

$$P_{\mathrm{FA}} = Q(\frac{\gamma - N\sigma_w^2}{\sqrt{2N\sigma_w^4}}) \tag{12}$$

$$P_{\mathrm{D}} = Q(\frac{\gamma - N(\sigma_w^2 + \sigma_s^2)}{\sqrt{2N(\sigma_w^2 + \sigma_s^2)^2}}) \tag{13}$$

The energy detector can meet any desired $P_{\mathrm{D}}$ and $P_{\mathrm{FA}}$ simultaneously if the number of samples index is not limited. Note that the energy detector can be applied to any of signal types. However, our study is to apply this detector to detect the digital TV.

### 3.2 Replica-Correlation Detector

A replica-correlation detector is performed based on the correlation between the received signal and the replica known primary signal. A decision value of replica-correlation detector is given by

$$T = \mathrm{Re}(\sum_{n=0}^{N-1} x[n]s^*[n]) > \gamma, \quad n = 0, 1, ...N-1 \tag{14}$$

The replica-correlation detector is depicted in the Figure 2. Again, for simplicity of derivation, we assume that the noise

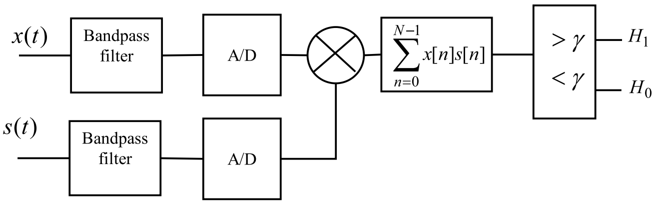


Figure 2 Replica-correlation detector

$w[n]$ and the signal $s[n]$ are independent. Since the received samples $x[n]$ is a linear combination of $w[n]$ and $x[n]$, therefore $x[n]$ is also independent. As the results, the $x[n]s[n]$ is a sequence of independent and identically distributed random variables with mean and variance

$$\mathrm{E}[x[n]s[n]|H_0] = 0 \tag{15}$$

$$\mathrm{var}[x[n]s[n]|H_0] = \sigma_w^2\sigma_s^2 \tag{16}$$

$$\mathrm{E}[x[n]s[n]|H_1] = \sigma_s^2 \tag{17}$$

$$\mathrm{var}[x[n]s[n]|H_1] = \sigma_s^2(1 + \sigma_w^2 + \sigma_s^2) \tag{18}$$

If the sample index $N$ is large, we can apply the central limit theorem to the decision static of the replica-correlation detector $T$, and then it can be approximated as a Gaussian random variable with mean and variance

$T \sim \mathcal{N}(0, N\sigma_w^2\sigma_s^2)$ under $H_0$

$T \sim \mathcal{N}(N\sigma_s^2, N\sigma_s^2(1 + \sigma_w^2 + \sigma_s^2))$ under $H_1$

Finally, the $P_{\mathrm{FA}}$ and $P_{\mathrm{D}}$ can be evaluated as

$$P_{\mathrm{FA}} = Q(\frac{\gamma}{\sqrt{N\sigma_w^2\sigma_s^2}}) \tag{19}$$

$$P_{\mathrm{D}} = Q(\frac{\gamma - N\sigma_s^2}{\sqrt{N\sigma_s^2(1 + \sigma_w^2 + \sigma_s^2)}}) \tag{20}$$

## 4. Simulation results

### 4.1 Energy detector simulation

In the energy detector, the optimal threshold can be computed when the $P_{\mathrm{FA}}$ is equal to the $P_{\mathrm{MD}}$ [5]. After mathematical simplification, the optimal threshold is given by $\gamma = \frac{2N\sigma_w^2(\sigma_w^2+\sigma_s^2)}{2\sigma_w^2+\sigma_s^2}$.

To evaluate the energy detector, we apply this detector to detect the digital TV (DVB). First, we plot the probability density function (pdf) of the distribution $T$ when the signal is absent ($H_0$) and when the signal is present $H_1$. It is shown in Figure 3. The energy detector decide the signal is present if the $T > \gamma$. In our case the optimal $\gamma$ is 33dB.

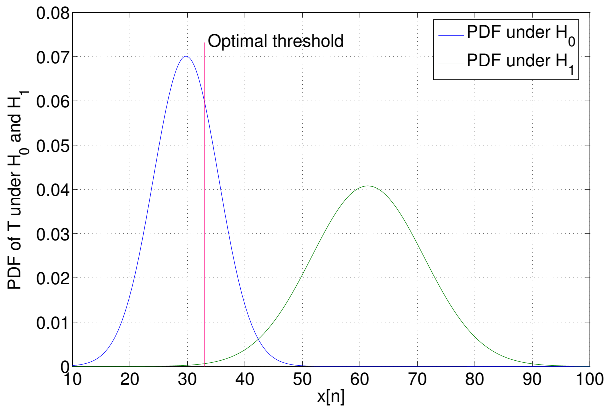


Figure 3 Energy detector: pdf

Secondly, we plot the probability of detection vs. the SNR by varying the $P_{\mathrm{FA}}$. It is given in Figure 4. Figure 4 shows that the probability of detection increase when $P_{\mathrm{FA}}$ also increase. Moreover, the performance of energy detector is excellent when the SNR is greater than $-5$ dB.

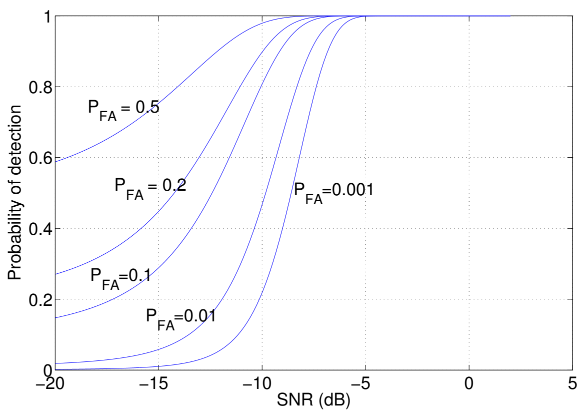


Figure 4 Energy detector: $P_{\mathrm{D}}$ vs. SNR

Thirdly, we plot the probability of detection vs. the probability of false alarm by varying the SNR, it is called the receiver operation characteristics. The result is shown in Figure 5. Again, if the probability of false alarm and probability of detection increase simultaneously and if the SNR greater that $-5$dB, regardless of the probability of false alarm the energy detector is optimum.

Lastly, we plot the probability of detection vs. sample index (window size). It is shown in Figure 6. We see that for each of SNR, the probability of detection increases with the number of sampling.

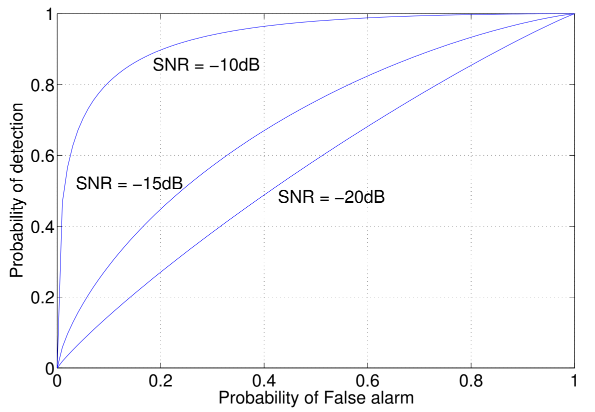


Figure 5 Energy detector: $P_{\mathrm{D}}$ vs. $P_{\mathrm{FA}}$

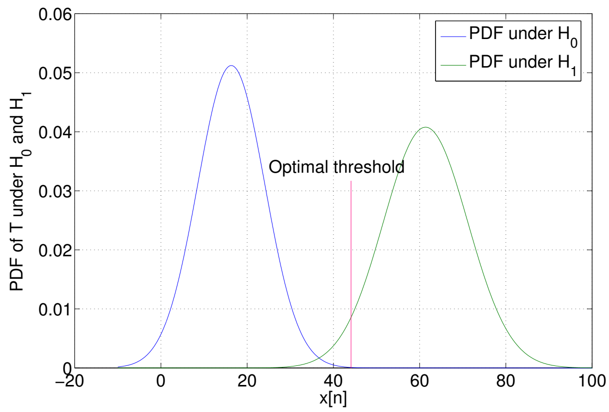


Figure 7 Replica-correlation detector: pdf

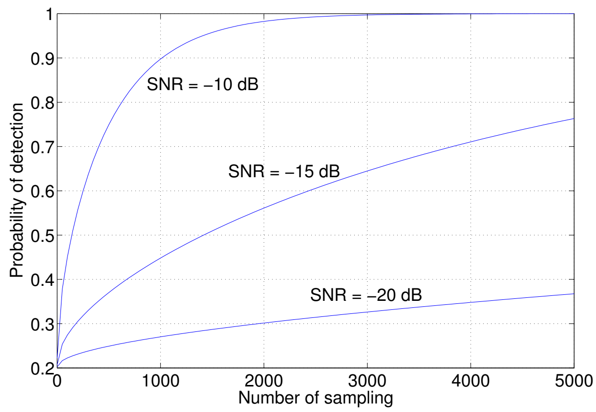


Figure 6 Energy detector: $P_{\mathrm{D}}$ vs. sample index

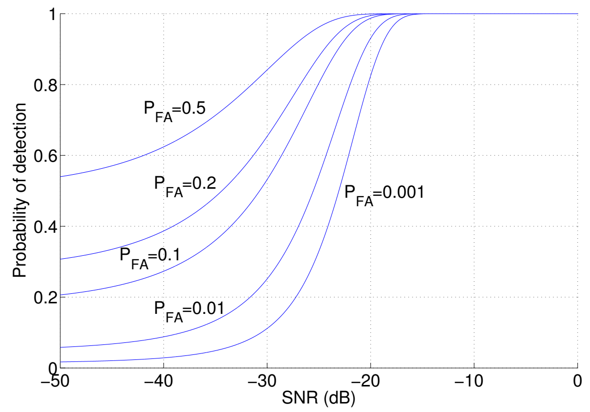


Figure 8 Replica-correlation detector: $P_{\mathrm{D}}$ vs. SNR

In summarize, we see that the energy detector can detect the signal at very low signal-to-noise ratio and its performance can be improve if the probability of false alarm or the samples index are increase. Moreover, as recommended in [7], if $P_{\mathrm{FA}} < 0.1$ and the $P_{\mathrm{D}} > 0.9$ are satisfied, and to use the energy detector the SNR should be not less than $-10$ dB.

### 4.2 Replica-correlation detector simulation

Again, the optimal threshold can be computed when the $P_{FA}$ is equal to the $P_{\mathrm{MD}}$. Therefore, the optimal threshold is given by $\gamma = \frac{N\sigma_s^2\sqrt{\sigma_w^2}}{\sqrt{1+\sigma_w^2+\sigma_s^2+\sqrt{\sigma_w^2}}}$.

We apply the replica-correlation detector to detect the digital TV (DVB). We also plot the probability density function of the distribution $T$ when the signal is absent ($H_0$) and when the signal is present $H_1$. It is shown in Figure 3. The energy detector decide the signal is present if the $T > \gamma$. In our case the $\gamma = 43$.

We also plot the probability of detection vs. the SNR by varying the $P_{\mathrm{FA}}$. Figure 8 shows that the replica-correlation detector can detect the DVB signal well when SNR is of $-15$ dB. Therefor, the replica-correlation detector is better than energy detector by reduction the SNR of $-10$ dB.

It is clear to see that the replica-correlation detector is much better than energy detector, by investigated the receiver operation characteristic of replica-correlation detector which is shown in Figure 9. The replica-correlation detector is optimum when the SNR is greater than $-15$dB.

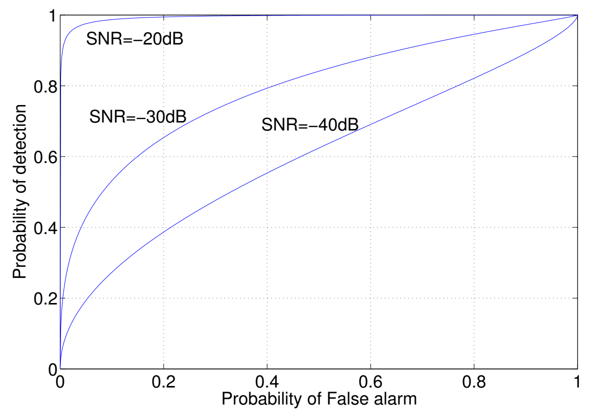


Figure 9 Replica-correlation detector: $P_{\mathrm{D}}$ vs. $P_{\mathrm{FA}}$

The same as energy detector, we plot the probability of detection vs. the sample index. In Figure 10, also shows that with the fixed value of SNR, the probability of detection increase with the number of sample index.

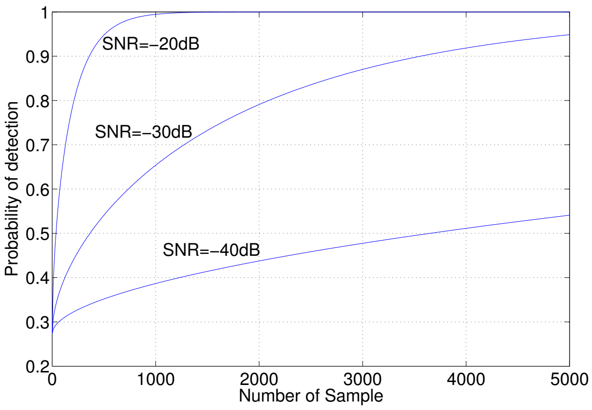


Figure 10 Replica-correlation detector: $P_{\mathrm{D}}$ vs. sample index

In summarize, we see that the performance of replica-correlation detector is better than energy detector. It can detect primary signal at lower SNR than energy detector. The same as energy detector, its performance can be improve if the probability of false alarm or the samples index are increase. Moreover, as recommended in [7], if $P_{\mathrm{FA}} < 0.1$ and the $P_{\mathrm{D}} > 0.1$ are satisfied, and to use the replica-correlation detector the SNR should be not less than $-20$ dB.

## 5. Future Work

We will further our study of the energy and replica-correlation detector for detection the analog TV (NTSC and PAL) and the digital TV (ISDB-T). Since both detectors require the knowledge of known characteristic of the primary signal which is difficult for the real application, we will also consider on the cyclostationary feature detection in our future works. Note that cyclostationary feature detection searches the unique cyclic frequency of different modulated signals. It does not require decoding of the primary signal and is robust to random noise and interference [6].

## 6. Conclusion

IEEE 802.22 system based on CR techniques operates in the TV bands without interference. Therefore, the CR must be able to identify the absence or the presence of the TV signals correctly. Since detection is the most important procedure of the CR, we have studied the energy detector and replica-correlation detector to detect the digital TV (DVB) signal. In our results show that the replica-correlation detection is better than energy detector with the reduction of SNR of $-10$dB.

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