

# Practical Detection Issues of Spectrum Sensing for Cognitive Radio System

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**Abstract** Spectrum sensing is an important functionality for secondary systems to ensure that they do not interfere primary systems when they try to access the unused spectrum of the primary systems. Energy detector is one of the possible candidates for spectrum sensing. In this paper, the important practical detection issues of spectrum sensing i.e. effect of quantization and effect of noise uncertainty on the performance of energy detector have been explored. Simulations have been carried out to show that both of these issues have significant effect on the overall performance of the detector. In addition, it has been shown that co-operative sensing helps to reduce those effects to some extent.

**Key words** Cognitive radio, spectrum sensing, energy detector, quantization, noise uncertainty, co-operative sensing

## 1. Introduction

The coexistence of primary and secondary systems on the same frequency band is possible only if the secondary system does not interfere the operation of the primary system. The secondary system has to perform continuous scanning of the spectrum of the primary system so that it can use the spectrum opportunistically. In this aspect, spectrum sensing is one of the most crucial functionalities of cognitive radio system.

Energy detector is one of the possible candidates of detectors for cognitive radio [1] not only because its implementation is simple but also because the cognitive radio receiver is likely to have very limited information about the signal of the primary system. Despite its simplicity, some of the detection issues that are inherent in cognitive radio receiver degrade its performance significantly. Two of such very important detection issues are the effect of quantization and the effect of noise uncertainty.

Although ADC is a very crucial element in all receivers, the effect of quantization in the detection performance is often neglected. The actual detection performance depends on the quantized samples which can deviate significantly from the performance estimated assuming the ideal samples with infinite precision. So, the authors tried to explore the detection performance taking into consideration the effect of

quantization and compared it with the performance based on the ideal samples.

Choosing the simplest model for background noise has been a trend for simplicity in analysis. However, real noise is a combination of noise from many different sources and its level is not exactly known. So, the difference in theoretical detection performance and the corresponding practical performance can be considerably high in many cases. For cognitive radio detectors also, most of the literatures assume that the background noise is additive white Gaussian and that its variance is exactly known, which is most of the time not the case in real environment. So, it is our motivation to explore the effect of noise uncertainty in the detection performance of energy detector.

In this paper, the effect of quantization on the detection performance of energy detector has been shown through simulations. The performance is compared with the case when ideal samples are assumed to indicate the amount of deviation quantization can result in the detection performance. The deviation in the detection performance because of the effect of noise uncertainty has been explored through simulations considering ideal as well as quantized samples. In addition, the effects are analyzed for the case of co-operative sensing between secondary users to examine the amount of improvement brought by cooperative sensing.

The rest of the paper is organized as following. Section 2

presents the concept of energy detector: non-cooperative and cooperative sensing. The theoretical background of quantization and noise uncertainty is presented in section 3. In section 4, the Integrated Services Digital Broadcasting - Terrestrial (ISDB-T) signal and simulation results are discussed. Section 5 is about conclusion and future work.

## 2. Energy Detector

Energy detector computes the energy of the received signal samples and compares it with the predefined threshold level to determine the presence/absence of the signal from the primary system. The description about single secondary user case (non-cooperative sensing) and multiple secondary users case (cooperative sensing) for energy detector follows.

### 2.1 Non-cooperative Sensing

Let us assume that the transmitted signal  $s[n]$  gets corrupted by the receiver noise  $w[n]$  which is complex additive white Gaussian with zero mean and variance  $\sigma_w^2$  [2]. The received signal  $x[n]$  will have either of the following two forms:

$$H_0 : x[n] = w[n] \quad \text{signal is absent,} \quad (1)$$

$$H_1 : x[n] = s[n] + w[n] \quad \text{signal is present,} \quad (2)$$

where  $n = 1, 2, \dots, N$  is the discrete time index and  $N$  is the number of samples considered.

The decision statistic of energy detector is given by

$$T_{\text{ncs}} = \sum_{n=1}^N x[n]x^*[n]. \quad (3)$$

Under both hypotheses,  $T_{\text{ncs}}$  follows chi-square distribution with  $2N$  degrees of freedom. With sufficiently large value of  $N$ , using central limit theorem, the distribution of the test statistic can be approximated as Gaussian. Hence the statistic is given by

$$\begin{cases} H_0 : T \sim \mathcal{N}(\mu_0, \sigma_0^2) \\ H_1 : T \sim \mathcal{N}(\mu_1, \sigma_1^2), \end{cases} \quad (4)$$

where  $\mathcal{N}(a, b)$  implies Gaussian distribution with mean  $a$  and variance  $b$ .

The mean and variance of the received signal are given by

$$\begin{cases} \mu_0 = N\sigma_w^2 \\ \sigma_0^2 = N\sigma_w^4 \end{cases} \quad (5)$$

and

$$\begin{cases} \mu_1 = N(\sigma_s^2 + \sigma_w^2) \\ \sigma_1^2 = N(\sigma_s^2 + \sigma_w^2)^2. \end{cases} \quad (6)$$

Now Substituting (5) and (6) in (4), the decision statistic can be formulated as

$$\begin{cases} H_0 : T \sim \mathcal{N}(N\sigma_w^2, N\sigma_w^4) \\ H_1 : T \sim \mathcal{N}(N(\sigma_s^2 + \sigma_w^2), N(\sigma_s^2 + \sigma_w^2)^2). \end{cases} \quad (7)$$

If required probability of false alarm  $P_{\text{FA}}$  is given, the threshold  $\gamma_{\text{ncs}}$  can be calculated as

$$\gamma_{\text{ncs}} = \sqrt{N\sigma_w^4}Q^{-1}(P_{\text{FA}}) + N\sigma_w^2. \quad (8)$$

Then, the probability of detection can be calculated as

$$P_D = Q\left(\frac{\gamma_{\text{ncs}} - N(\sigma_w^2 + \sigma_s^2)}{\sqrt{N(\sigma_w^2 + \sigma_s^2)^2}}\right). \quad (9)$$

### 2.2 Cooperative Sensing

In cooperative sensing, the secondary users share the information gathered by each unit about the primary signal among each other. An arbitrary scenario of cooperative sensing assuming  $N_u$  secondary users is shown in Fig. 1. Each of the secondary users get a fraction of the total signal from the primary system transmitter. The fractions  $\beta_1, \beta_2, \dots, \beta_{N_u}$  are the functions of various factors such as the distance of the respective secondary user from the primary system transmitter, gain of the secondary user etc.

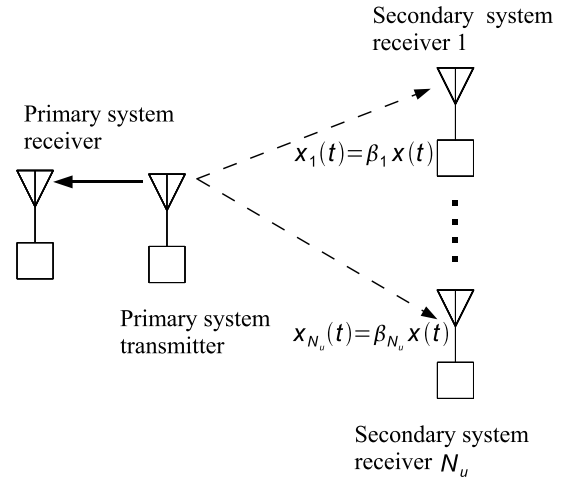


Figure 1 Cooperative Sensing Scenario

In this case, the statistical test [3] can be expressed as

$$T_{\text{cs}} = \sum_{i=1}^{N_u} \sum_{n=1}^N x_i[n]x_i^*[n]. \quad (10)$$

Now with similar analysis as in case of non-cooperative sensing, the threshold  $\gamma_{\text{cs}}$  can be written as

$$\gamma_{\text{cs}} = \sqrt{N \sum_{i=1}^{N_u} \sigma_{w,i}^4}Q^{-1}(P_{\text{FA}}) + N \sum_{i=1}^{N_u} \sigma_{w,i}^2 \quad (11)$$

and the probability of detection can be formulated as

$$P_D = Q\left(\frac{\gamma_{\text{cs}} - N \sum_{i=1}^{N_u} (\sigma_{w,i}^2 + \sigma_{s,i}^2)}{\sqrt{N \sum_{i=1}^{N_u} (\sigma_{w,i}^2 + \sigma_{s,i}^2)^2}}\right). \quad (12)$$

### 3. Effect of Quantization and Noise Uncertainty on Detection Performance of Energy detector

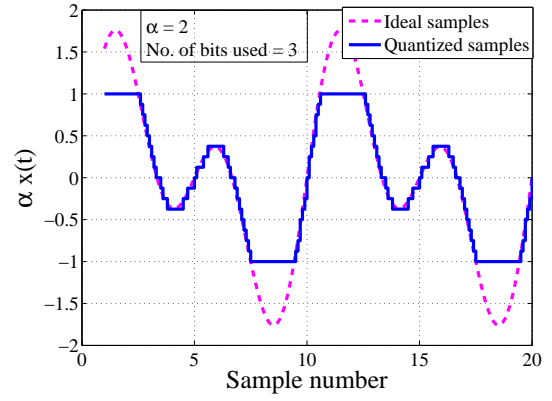
#### 3.1 Quantization

Quantization is an inseparable and very important aspect of practical systems. It is obvious that quantization should make detection harder. So, a quantizer is often thought of as an additional source of noise. More specifically stating, quantizing the signal effectively reduces the SNR at the receiver which means that it yields an effective SNR loss in detection performance.

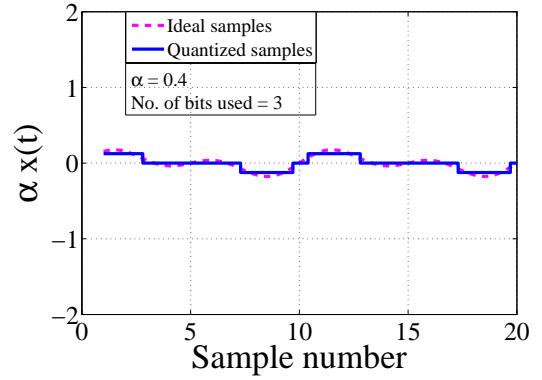
The quantization process can cause two kinds of errors: quantization error and clipping error. Generally, an automatic gain controller (AGC) is used to control the level of the input signal to the ADC. However, if the AGC is not designed for optimum performance, both of these errors can significantly degrade the detection performance.

Our primary concern is to detect extremely weak signal from the primary system in negative SNR region. The incoming signal is generally amplified so that the weak signal is detectable. However, this may lead to clipping of the signal when it is sampled by the ADC. For signals like OFDM that have quite high peak to average power ratio (PAPR), the clipping error will cover a considerable fraction of the signal level. Especially for negative SNR case, clipping is a very critical problem to detect the weak signal from the primary system. In order to avoid clipping, a high ADC back-off margin can be set by decreasing the scaling factor ( $\alpha$ ) of the AGC. This can guarantee that clipping will not occur. However, this solution reduces the effective number of bits used for quantization and consequently the quantization error will increase. So, with this strategy, though clipping error is controlled, quantization error will play the key role to degrade the detection performance. An arbitrary example is shown in Fig. 2 to demonstrate the effect of clipping and quantization errors. In Fig. 2(a), for a sinusoidal signal  $x(t)$ , the AGC scaling factor used is  $\alpha = 2$  while the full scale range (FSR) of the ADC is  $\pm 1$ . This results the clipping error to be quite high and is dominant error in this case. On the other hand, in Fig. 2(b),  $\alpha$  is taken as 0.4 to avoid clipping error. There is no clipping this time but the signal level is so small that the quantization error becomes relatively very high. So, it is very important to clarify the optimum values/range of  $\alpha$  for fairly well detection performance.

Figure 3 shows the low pass model of the receiver based on energy detector. The out of band components in the received signal are filtered out by the low pass filter. Let us assume that the filter, which is ideal, limits noise power without any influence on the signal. All blocks of the receiver except



(a) clipping.



(b) Quantization.

Figure 2 Effect of clipping and quantization errors.

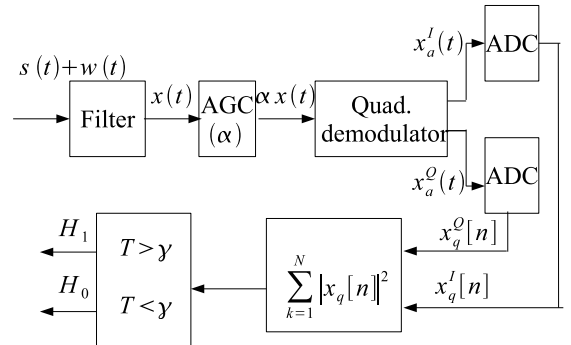


Figure 3 Model of the Receiver

ADCs are assumed to be linear and the sampling time is assumed to have no jitter [4]. The signal at the filter output can be expressed as

$$x(t) = s(t) + w(t), \quad (13)$$

where  $s(t)$  is the signal component and  $w(t)$  is the noise component which is a complex Gaussian variable with zero mean and variance  $\sigma_w^2$ .

Then the resulting signal is scaled by AGC scaling factor  $\alpha$ . The output of the AGC is fed to the quadrature demodulator. Then the outputs are sampled and quantized by the ADC and then subjected to hypothesis test in order to find

out whether the signal from the primary system is present or absent. The dynamic range of the ADC depends on its full scale range (FSR). Let us assume that the FSR of the ADC is equal to its average input level. Now, if the input to the ADC be normalized with respect to its average voltage, the FSR of the ADC can be taken as  $\pm 1$  which makes the analysis simpler.

The Inphase (I) and quadrature phase (Q) ADC inputs are represented as

$$\begin{cases} x_a^I(t) = \frac{1}{\sqrt{2}} \Re[\alpha x(t)] \\ x_a^Q(t) = \frac{1}{\sqrt{2}} \Im[\alpha x(t)]. \end{cases} \quad (14)$$

The I and Q outputs of the ADC at  $t = nT_q$  are given by,

$$\begin{cases} x_q^I[n] = q(x_a^I(nT_q)) \\ x_q^Q[n] = q(x_a^Q(nT_q)), \end{cases} \quad (15)$$

where  $T_q$  is the sampling interval of each of the ADCs.

The general form for the quantized output may be written as

$$q(x) = \begin{cases} 1 & x \geq 1 \\ \lfloor \frac{x}{\Delta_q} \rfloor \Delta_q + \frac{\Delta_q}{2} & -1 \leq x < 1 \\ -1 & \text{otherwise} \end{cases} \quad (16)$$

where  $\Delta_q$  is the stepsize of quantization given by

$$\Delta_q = \frac{2}{2^M - 1}, \quad (17)$$

where  $M$  is the number of bits used for quantization.

Now, the distortion caused by the ADC can be expressed as following

$$\begin{cases} e_q^I[n] = x_a^I(nT_q) - x_q^I[n] \\ e_q^Q[n] = x_a^Q(nT_q) - x_q^Q[n]. \end{cases} \quad (18)$$

The nature of distortion depends on the level of the ADC input. If the amplitude of the input to the ADC is within the FSR as shown in (19), the magnitude of distortion (quantization error) is upper limited as shown in (20).

$$|x_a(nT_q)| < 1 + \frac{\Delta_q}{2} \quad (19)$$

$$|e_q[k]| < \frac{\Delta_q}{2} \quad (20)$$

On the other hand, when the ADC input amplitude is out of the FSR i.e.

$$|x_a(nT_q)| > 1 + \frac{\Delta_q}{2}, \quad (21)$$

the error is given by

$$|e_q[n]| > \frac{\Delta_q}{2}. \quad (22)$$

In order to evaluate the degradation caused by quantization in the detection performance, the parameters of the

ideal samples  $x[n]$  should be replaced by that of the quantized samples  $q(x)$  in (9) and (12). As the quantization error depends on the number of bits used for quantization ( $M$ ) and the value of AGC scaling factor ( $\alpha$ ), it is obvious that the performance of the detector assuming quantized samples also depends on  $M$  and  $\alpha$ .

### 3.2 Uncertainty Model for Energy Detector

Although it is generally assumed for simplicity that the variance of the receiver noise is known, noise variance is never exactly known in case of real systems even if the system is calibrated. There are several factors that contribute for the existence of noise uncertainty. For example, thermal noise due to change in temperature, change in amplifier gain due to change in temperature, calibration error etc. As noise uncertainty in the receiver is unavoidable, it is very important to analyze its effect on detection performance.

Let us model the noise process  $w[n]$  to have any distribution  $W$  from a set of possible distributions  $\mathcal{W}$ . This set is called the noise uncertainty set. Although the actual noise variance might vary over distributions in the set  $\mathcal{W}$ , let us assume that there is a single nominal noise variance  $\sigma_n^2$  associated with the noise uncertainty set  $\mathcal{W}$ .

As energy detector evaluates the detection performance based on energy of the incoming signal, the distributional uncertainty of noise can be summarized in a single interval  $\sigma_w^2 \in [(\frac{1}{\rho})\sigma_n^2, \rho\sigma_n^2]$  where  $\sigma_n^2$  is the nominal noise power and  $\rho > 1$  is a parameter that quantifies the size of the noise uncertainty. This parameter is often considered in its dB equivalent as  $10\log_{10}(\rho)$ .

Including the effect of noise uncertainty, (8) and (9) can be modified for non-cooperative sensing as described in [5] as

$$\gamma_{\text{ncs}} = \sqrt{N\rho^2\sigma_n^4}Q^{-1}(P_{\text{FA}}) + N\rho\sigma_n^2, \quad (23)$$

$$P_D = Q\left(\frac{\gamma_{\text{ncs}} - N(\frac{1}{\rho}\sigma_n^2 + \sigma_s^2)}{\sqrt{N(\frac{1}{\rho}\sigma_n^2 + \sigma_s^2)^2}}\right). \quad (24)$$

Similarly for cooperative sensing, (11) and (12) can be modified as

$$\gamma_{\text{cs}} = \sqrt{N\rho^2 \sum_{i=1}^{N_u} \sigma_{n,i}^4}Q^{-1}(P_{\text{FA}}) + N\rho \sum_{i=1}^{N_u} \sigma_{n,i}^2, \quad (25)$$

$$P_D = Q\left(\frac{\gamma_{\text{cs}} - N \sum_{i=1}^{N_u} (\frac{1}{\rho}\sigma_{n,i}^2 + \sigma_{s,i}^2)}{\sqrt{N \sum_{i=1}^{N_u} (\frac{1}{\rho}\sigma_{n,i}^2 + \sigma_{s,i}^2)^2}}\right). \quad (26)$$

## 4. Simulation

### 4.1 ISDB-T Signal

As the current discussions about cognitive radio system are focused mainly on TV system as the primary user, the Integrated Services Digital Broadcasting - Terrestrial (ISDB-T)

Table 1 Parameters of the ISDB-T signal used for simulation

Parameters	Values
Effective symbol length ( $T_u$ )	252 $\mu$ s
Guard interval ( $T_g$ )	$\frac{T_u}{4} = 63 \mu$ s
Symbol duration ( $T_s$ )	315 $\mu$ s
Number of symbols	20
Total duration	6.3 ms
Number of carriers	1405
Total bandwidth	5.575 MHz
Sampling frequency	16.254 MHz

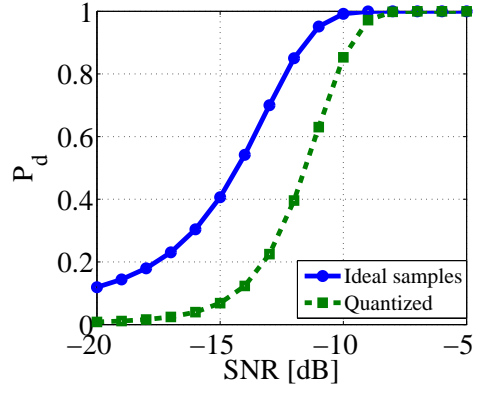
signal was generated following the description given in [6] for Mode 1. The parameters of the generated signal are given in Table 1. The PAPR of the generated ISDB-T signal is 11.16 dB.

#### 4.2 Simulation Results

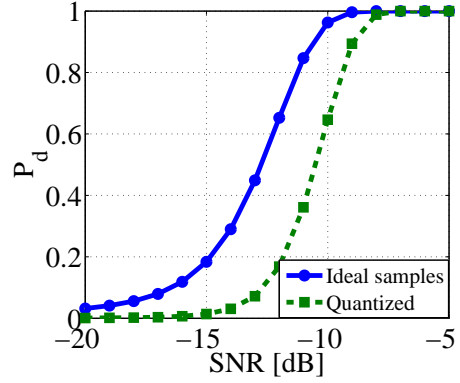
Figures 4(a) and (b) show the detection performance of energy detector with and without taking into consideration the effect of quantization for non-cooperative sensing taking  $P_{FA}$  as 5% and 1% respectively. As quantization is an extra source of noise, degradation occurs in the detection performance with quantized samples. For example, if ideal samples are considered, the minimum required value of SNR for 100% probability of detection is about  $-10$  dB for Fig. 4(a) and about  $-9$  dB for Fig. 4(b). On the other hand, when quantized samples are considered, the minimum required value of SNR is about  $-8$  dB and  $-7$  dB respectively.

Figure 5 shows the effect of quantization noise and clipping for different values of  $M$  for non-cooperative sensing taking  $P_{FA}$  equal to 1% for SNR =  $-9$  dB. The performance is optimum within certain range of  $\alpha$ . For lower values of  $\alpha$ , the probability of detection can be improved by increasing the number of bits of ADC because in this case quantization error is dominant. On the other hand, for higher values of  $\alpha$ , increasing the number of bits does not help because the clipping noise becomes dominant in this case. The optimum values of  $\alpha$  for different values of  $M$  from Fig. 5 are shown in Table 2.

Figures 6(a) and (b) show the performance of energy detector for different values of noise uncertainty  $\rho$  assuming ideal samples i.e. using (24) and (26) for  $P_{FA} = 5\%$  and  $P_{FA} = 1\%$  respectively. The  $\rho = 0$  curves in the figure are the curves considering ideal samples and without noise uncertainty. So, they show the best performance. As the value of  $\rho$  increases, the performance degrades. In order to demonstrate the effect of cooperative sensing, 2 secondary users are assumed. It can be clearly seen from the figure that cooperative sensing improves the performance. However the improvement brought by cooperative sensing diminishes with increase in noise uncertainty.



(a)  $P_{FA} = 5\%$ .



(b)  $P_{FA} = 1\%$ .

Figure 4 Detection performance with ideal samples and with quantized samples (Sensing time ( $T$ ) =  $63\mu$ s,  $M = 8$ ,  $\alpha = 1$ ).

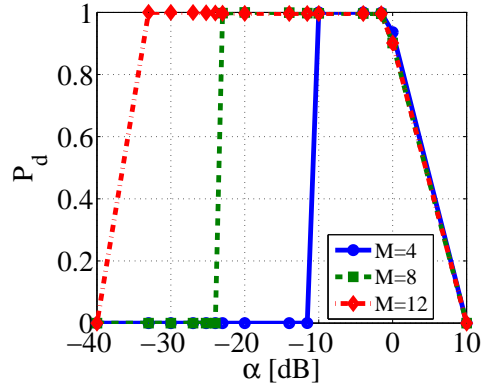
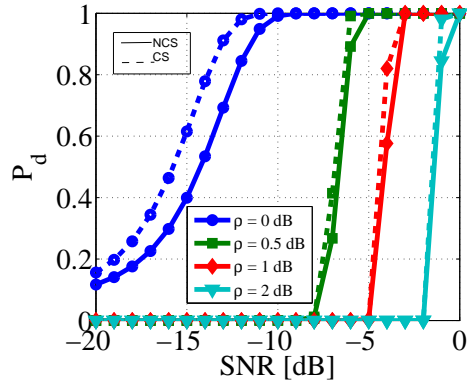
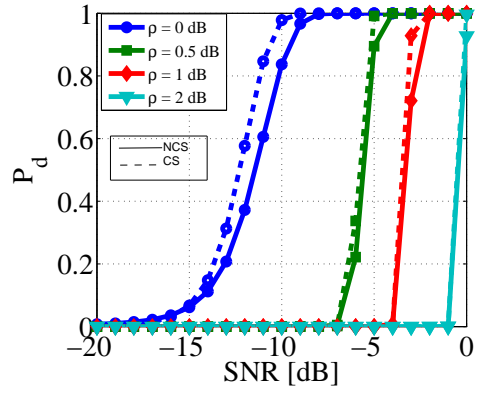


Figure 5 Detection performance w.r.t. AGC scaling factor (Sensing time ( $T$ ) =  $63\mu$ s,  $P_{FA} = 1\%$ , SNR =  $-9$  dB).

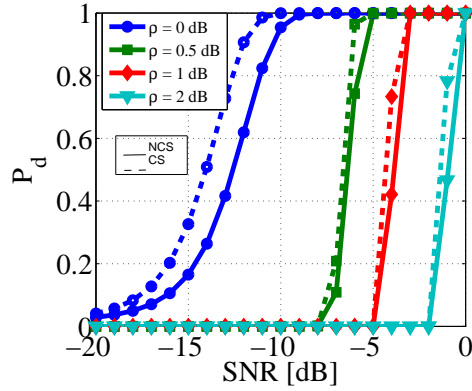
Figures 7(a) and (b) show the performance of energy detector for different values of noise uncertainty  $\rho$  assuming quantized samples for  $P_{FA} = 5\%$  and  $P_{FA} = 1\%$  respectively. With increasing value of  $\rho$ , the performance degrades. Compared to Figures 6(a) and (b), the performance is more degraded. It is clear because the overall degradation is due to the combined effect of quantization and noise uncertainty. In this case also, cooperative sensing improves the perfor-



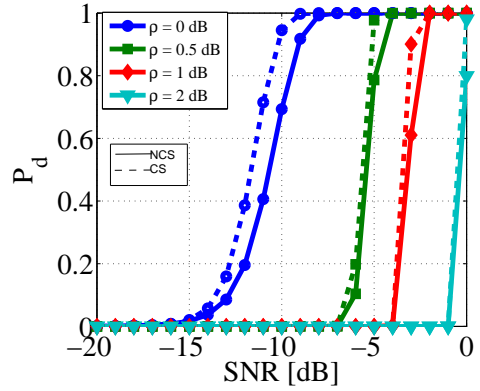
(a)  $P_{FA} = 5\%$ .



(a)  $P_{FA} = 5\%$ .



(b)  $P_{FA} = 1\%$ .



(b)  $P_{FA} = 1\%$ .

Figure 6 Detection Performance for different values of  $\rho$  taking ideal samples (Sensing time ( $T$ ) =  $63\mu s$ ,  $\alpha = 1$ ).

Figure 7 Detection Performance for different values of  $\rho$  taking quantized samples (Sensing time ( $T$ ) =  $63\mu s$ ,  $\alpha = 1$ ).

Table 2 Optimum range of  $\alpha$  based on  $M$

$M$	$\alpha_{opt}$
4	-10 to -2 dB
8	-24 to -2 dB
12	-33 to -2 dB

mance but the improvement brought by cooperative sensing diminishes with increase in noise uncertainty.

## 5. Conclusion and Future Work

### 5.1 Conclusion

Although cognitive radio possesses huge potential to enable effective use of vast amount of underutilized spectrum, practical detection issues must be seriously considered so that the effective detection performance can be maintained. In this paper, quantization as well as noise uncertainty have been shown to induce considerable amount of degradation in the detection performance of energy detector. It has also been shown that co-operative sensing helps to reduce the effect of both of these factors in the overall detection performance. However, with increase in noise uncertainty of the receiver, the contribution of cooperative sensing dims.

### 5.2 Future Work

Though energy detector is suitable as a cognitive radio receiver, its performance is easily rendered by the detection issues to great extent. Cyclostationary detector has been shown to work far superior to energy detector provided that the secondary system has the knowledge of the signal from the primary system to some extent. Although it is impractical to assume that the cognitive radio detector knows everything about the signal from the primary system, it is still reasonable to assume that it has the knowledge about at least some very fundamental parameters of the primary signal such as its bandwidth, carrier, modulation scheme etc. With this assumption, the authors intend to explore about the effect of the practical detection issues dealt with in this paper for cyclostationary detector as well.

### References

- [1] S. Haykin, "Cognitive radio: Brain empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, Vol. 23, No. 2, pp.201-220, Feb. 2005.
- [2] S. Kay, "Statistical decision theory I (Ch. 3)" and "Deterministic signals (Ch. 4)," in *Fundamentals of Statistical Signal Processing, Detection Theory*, pp.60-140, Prentice Hall, 1998.
- [3] H. Uchiyama, K. Umebayashi, T. Fujii, F. Ono, K. Sak-

- aguchi, Y. Kamiya and Y. Suzuki, "Study on Soft Decision Based Cooperative Sensing for Cognitive Radio Networks," IEICE Trans. Commun., Vol.E91-B, No.1, Jan. 2008.
- [4] M. Sawada, H. Okada, T. Yamazato and M. Katayama "Influence of ADC nonlinearity on the performance of an OFDM receiver," IEICE Trans. Commun., Vol.E89-B, No.12, Dec. 2006.
- [5] R. Tandra and A. Sahai, "SNR Walls for Signal Detection," IEEE Journal of Selected Topics in Signal Processing, Vol. 2, No. 1, Feb. 2008.
- [6] "Transmission System for Digital Terrestrial Television Broadcasting," ARIB Standard, ARIB STD-B31 Version 1.5, Association of Radio Industries and Businesses, 2003.