

Performance of MUSIC and ESPRIT for Joint Estimation of DOA and Angular Spread in Slow Fading Environment

Jung-Sik JEONG[†], Kei SAKAGUCHI[†], Jun-ichi TAKADA[†],
and Kiyomichi ARAKI[†], *Regular Members*

SUMMARY It is known that MUSIC and ESPRIT algorithms can estimate simultaneously both the instantaneous Direction of Arrival (DOA) and the instantaneous Angular Spread (AS) in multiple scattering environments. These algorithms use the Extended Array Mode Vector (EAMV) with complex angle. The previous work evaluated the performance of those algorithms by comparing the estimated DOA and the estimated AS with the DOA and the AS given in the EAMV, which uses the first-order approximation. Thus, this evaluation method has not clearly reflected the estimation accuracy of MUSIC and ESPRIT. This paper presents the joint estimation performance of MUSIC and ESPRIT by introducing the criteria for evaluation. For this, the spatial signature (SS) is reconstructed from the estimates of the DOA and the AS, and compared to the true SS in the meaning of data fitting.

key words: DOA, angular spread, local scattering, MUSIC, ESPRIT

1. Introduction

In multiple scattering environments, the Angular Spread (AS) causes errors to the data model based on the plane wave in array antenna signal processing. This deteriorates the performance of superresolution algorithms for only the Direction of Arrival (DOA) estimation, such as MUSIC [1] and ESPRIT [2]. In recent years, the problem of estimating both the DOA and the AS of signals has received considerable attention in mobile communication system [3]–[7]. In [6], [7], MUSIC and ESPRIT were used in order to jointly estimate the DOA and the AS in a slow fading channel. These approaches use the Extended Array Mode Vector (EAMV) that may represent the instantaneous AS as well as the instantaneous DOA given during one data burst. To illustrate the performance of MUSIC and ESPRIT for the joint estimation, References [6], [7] have compared the estimates obtained from MUSIC and ESPRIT with the DOA and the AS given in the EAMV. However, since the EAMV uses the first order approximation, the evaluation method in [6], [7] does not show clearly the performance of MUSIC and ESPRIT for the joint estimation.

This paper presents the joint estimation performance of MUSIC and ESPRIT by introducing the criteria for evaluation. For this, the spatial signature (SS) is

reconstructed from the estimates of the DOA and the AS, and evaluated by the criteria that we introduce here. The estimated SS is compared to the true SS in the meaning of the data fitting error, which may represent the proximity of the estimated SS with respect to the true SS. As a practical problem, the SS reconstructed from the DOA and AS estimates may be effectively utilized to recover signal waveform and to maximize Signal-to-Interference plus Noise Ratio (SINR) in the slow fading environments [8]–[11], in which the SS is assumed to be constant during one data burst. It has shown [8] that when both the DOA and the AS are utilized to combine the received signals, the array output SINR can be considerably improved as compared to the case of using the DOA only.

This paper is organized as follows. In Sect. 2, the data model for the received signals and the EAMV are described. MUSIC and ESPRIT for the joint estimation are discussed in Sect. 3. The simulation results followed by conclusions are presented in Sects. 4 and 5, respectively.

2. Data Model and Extended Array Mode Vector

In local scattering model [12], as shown in Fig. 1, it is assumed that there is no line-of-sight between the Mobile Station (MS) and the Base Station (BS). The received signal from the MS to the BS is modeled as a superposition of a large number of independent and identically distributed (i.i.d.) plane waves, which consist of coherent multipath waves.

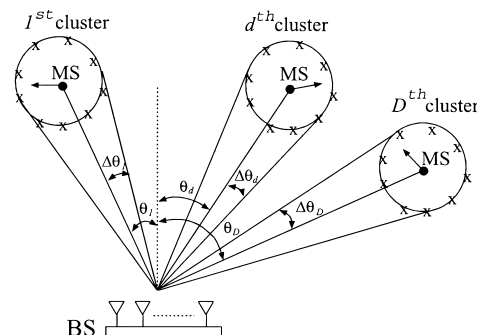


Fig. 1 Multiple clusters modeled by local scatterers.

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[†]The authors are with the Faculty of Engineering, Tokyo Institute of Technology, Tokyo, 152-8552 Japan.

Assume that D narrowband emitter signals, $s_d(t)$, $d = 1, \dots, D$, impinge on the array antenna with L elements of the BS and all the time delays of scattered waves are small in comparison with the symbol duration time. The received signal vector at the BS, $\mathbf{x} \in \mathbb{C}^{L \times 1}$, is modeled by

$$\begin{aligned} \mathbf{x}(t) &= \sum_{d=1}^D \sum_{k=1}^{K_d} \beta_{dk} \mathbf{a}(\theta_d + \tilde{\theta}_{dk}) s_d(t) + \mathbf{n}(t) \\ &= \mathbf{V} \mathbf{s}(t) + \mathbf{n}(t) \end{aligned} \quad (1)$$

where

$$\mathbf{s}(t) = [s_1(t), \dots, s_D(t)]^T \in \mathbb{C}^{D \times 1},$$

$$\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_D] \in \mathbb{C}^{L \times D},$$

$$\mathbf{v}_d = \sum_{k=1}^{K_d} \beta_{dk} \mathbf{a}(\theta_d + \tilde{\theta}_{dk}) \in \mathbb{C}^{L \times 1} : \text{spatial signal-} \\ \text{ture characterizing mobile channel,}$$

$$\mathbf{n}(t) \in \mathbb{C}^{L \times 1} : \text{additive white Gaussian noise vector,}$$

$$K_d : \text{total number of scatterers around the } d\text{-th MS,}$$

$$\beta_{dk} : \text{multipath coefficient with complex amplitude of the } k\text{-th scattered signal with respect to the } d\text{-th MS,}$$

$$\mathbf{a}(\theta) \in \mathbb{C}^{L \times 1} : \text{array mode vector with DOA } \theta,$$

$$\theta_d + \tilde{\theta}_{dk} : \text{arrival angle of the } k\text{-th scattered signal with respect to the } d\text{-th MS.}$$

As described in [6], using the first-order approximation with respect to $\tilde{\theta}_{dk}$, it is known that $\mathbf{v}_d$, $d = 1, \dots, D$, can represent the AS as well as the DOA given at a time instant of fading under the two main assumptions, namely, (i) the slow fading environments, (ii) the smaller AS as compared with the 3 dB beamwidth of the array antenna. In the slow fading channel, the SS is assumed to be constant during the observation period in one data burst. As a result, the SS is approximated by

$$\mathbf{v}_d \simeq \gamma_d \mathbf{a}(\vartheta_d) = \gamma_d \mathbf{a}(\theta'_d + j\Delta\theta_d), \quad d = 1, \dots, D, \quad (2)$$

where

$$\gamma_d = \sum_{k=1}^{K_d} \beta_{dk} : \text{Fading coefficient;}$$

$$\xi_d = \sum_{k=1}^{K_d} \beta_{dk} \tilde{\theta}_{dk} / \sum_{k=1}^{K_d} \beta_{dk} = \xi_{dr} + j\xi_{di} \\ : \text{Spread parameter;}$$

$$\theta'_d = \theta_d + \xi_{dr} : \text{Instantaneous DOA} \\ \text{perturbed by fading;}$$

$$\vartheta_d = \theta'_d + j\Delta\theta_d : \text{Complex angle;}$$

$$\Delta\theta_d = \xi_{di} : \text{Instantaneous AS;}$$

$\mathbf{a}(\vartheta_d)$ in Eq. (2) is defined as the Extended Array Mode Vector (EAMV), where $\vartheta_d = \theta'_d + j\Delta\theta_d$.

Assume that the noise for each array element is uncorrelated with the signals, spatially and temporally white, and has its variance σ_n^2 . The ensemble averaged covariance matrix of $\mathbf{x}(t)$, $\mathbf{R} \in \mathbb{C}^{L \times L}$, is given as

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{V} \mathbf{R}_s \mathbf{V}^H + \sigma_n^2 \mathbf{I} \quad (3)$$

where $(\cdot)^H$ denotes Hermitian transpose operation, $\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)] \in \mathbb{C}^{D \times D}$ is the covariance matrix of the signals, and $\mathbf{I} \in \mathbb{R}^{L \times L}$ is an identity matrix. Given N samples, $\mathbf{R}$ can be estimated by

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}(t_n)\mathbf{x}^H(t_n). \quad (4)$$

Throughout this paper, $\hat{(\cdot)}$ denotes the estimated value of $(\cdot)$.

3. Joint Estimation of DOA and AS

The existence of angular spreading causes errors to the data model based on the plane wave in array signal processing. This deteriorates the performance of conventional superresolution algorithms for only the Direction of Arrival (DOA) estimation, such as MUSIC [1] and ESPRIT [2]. In [6], [7], MUSIC and ESPRIT based on the EAMV were used in order to jointly estimate both the DOA and the AS. As a result, these approaches may reduce the effect of modeling error by estimating the AS as well as the DOA. In this section, MUSIC and ESPRIT algorithms for the joint estimation are briefly summarized.

3.1 MUSIC

Using the noise subspace and the EAMV, the cost function of MUSIC is defined by

$$f(\theta, \Delta\theta) = \frac{\|\mathbf{E}_N^H \mathbf{a}(\vartheta)\|^2}{\|\mathbf{a}(\vartheta)\|^2} \quad (5)$$

where $\mathbf{E}_N \in \mathbb{C}^{L \times (L-D)}$ denotes the noise subspace, $\vartheta \equiv \theta + j\Delta\theta$, and $\|\cdot\|$ denotes the Euclidean norm. Thus, MUSIC finds D pairs, $\{\hat{\theta}_d, \hat{\Delta\theta}_d\}_{d=1}^D$, which correspond to the local minima of $f(\theta, \Delta\theta)$ by searching $\mathbf{a}(\vartheta)$ in the complex angle space. Here, it should be noted that Eq. (5) is the same as that of the conventional MUSIC when $\Delta\theta = 0$.

3.2 ESPRIT

The ESPRIT algorithm requires the array antenna with the structure that can be decomposed into two identical subarrays. The corresponding elements of the two subarrays are displaced from each other by a fixed translational distance, Δ in wavelength. Then, the array

should satisfy a translational invariance principle [2]. Firstly, we consider the one signal case. The size of each subarray is assumed to be $L - 1$. The generalized expression of $\mathbf{v}_d^{(i)}$, $d = 1, \dots, D$, $i = 0, 1$ for two subarrays can be obtained as follows.

$$\begin{aligned} \mathbf{v}_d^{(0)} &= \sum_{k=1}^{K_d} \beta_{dk} \mathbf{a}(\theta_{dk}) \\ &\simeq \gamma_d \mathbf{a}(\vartheta_d), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathbf{v}_d^{(1)} &= \sum_{k=1}^{K_d} \beta_{dk} e^{j2\pi\Delta \sin(\theta_d + \tilde{\theta}_{dk})} \mathbf{a}(\theta_d + \tilde{\theta}_{dk}) \\ &\simeq \gamma_d e^{j2\pi\Delta \sin(\vartheta_d)} \mathbf{a}(\vartheta_d) \end{aligned} \quad (7)$$

where $\mathbf{v}_d^{(i)}$, $i = 0, 1$ denotes the SS related to the d -th MS with respect to the i -th subarray. From this, it is clear that the translational invariance principle can also be utilized as in the case of plane wave signal model, so that the estimate of $\vartheta_d = \theta_d + j\Delta\theta_d$ can be obtained. For D signals, the translational invariance principle may be extended. Thus, when the signal subspace is estimated from the observation data, we have resultantly

$$\mathbf{J}_1 \hat{\mathbf{E}}_S = \mathbf{J}_2 \hat{\mathbf{E}}_S \mathbf{\Psi} \quad (8)$$

where $\mathbf{J}_1 = [\mathbf{I}_{L-1} \ \mathbf{0}] \in \mathbb{R}^{(L-1) \times L}$, $\mathbf{J}_2 = [\mathbf{0} \ \mathbf{I}_{L-1}] \in \mathbb{R}^{(L-1) \times L}$, $\mathbf{0}$ is L -dimensional zero vector, and $\mathbf{I}_{L-1} \in \mathbb{R}^{(L-1) \times (L-1)}$ is the identity matrix, $\hat{\mathbf{E}}_S \in \mathbb{C}^{L \times D}$ denotes the estimated signal subspace, and $\mathbf{\Psi} \in \mathbb{C}^{D \times D}$. Thus, ESPRIT finds the (total) least squares solution in Eq. (8), $\hat{\mathbf{\Psi}}$. As a result, letting $\{\hat{\mu}_d, d = 1, \dots, D\}$ be the eigenvalues of $\hat{\mathbf{\Psi}}$, the estimates of the DOA and the AS are obtained from the relation of the phase delays and the amplitude variations between the two subarrays, i.e., $e^{j2\pi\Delta \sin \hat{\vartheta}_d} = \hat{\mu}_d$, $d = 1, \dots, D$. Thus,

$$\hat{\vartheta}_d = \hat{\theta}'_d + j\Delta\hat{\theta}_d = \sin^{-1} \frac{\log \hat{\mu}_d}{j2\pi\Delta} \quad d = 1, \dots, D. \quad (9)$$

For $\Delta\hat{\theta}_d = 0$, the magnitude of the estimated complex number $\hat{\mu}_d$ equals to unity. Thus, $\hat{\theta}_d = \sin^{-1} \frac{\arg \hat{\mu}_d}{2\pi\Delta}$. In the scattering environments, however, $|\hat{\mu}_d|$ will be largely deviated from unity as the AS increases. In the sequel, introducing the EAMV with complex angle to the ESPRIT[†] we can estimate both the DOA and the AS.

3.3 Relations with Subspace Fitting

In Eq. (2), it is impossible to individually estimate $\tilde{\theta}_{dk}$ and β_{dk} ($k = 1, \dots, K_d$) observed at a time instant. Instead of that, we know from Sects. 3.1 and 3.2 that the DOA and the AS can be estimated by using the proposed MUSIC. In fact, the problem of jointly estimating the DOA and the AS may be interpreted as the problem of fitting the observed data as follows. Firstly, for

one signal case, this problem is given by

$$[\hat{\vartheta}, \hat{\gamma}] = \arg \min_{\vartheta, \gamma} \|\mathbf{v} - \gamma \mathbf{a}(\vartheta)\|^2 \quad (10)$$

where $\mathbf{v}$ is the SS. Equation (10) finds the best parameters $\hat{\vartheta}$, $\hat{\gamma}$ that are closest to $\mathbf{v}$. Given $\mathbf{a}(\vartheta)$, the least square solution $\hat{\gamma}$ can be obtained by

$$\hat{\gamma} = \frac{\mathbf{a}(\vartheta)^H \mathbf{v}}{\|\mathbf{a}(\vartheta)\|^2}. \quad (11)$$

Substituting $\hat{\gamma}$ into Eq. (10), we resultantly obtain the solution,

4. Computer Simulations

To evaluate the joint estimation performance of MUSIC and ESPRIT, the EAMV is reconstructed from the estimates of both the DOA and the AS, and then the SS is estimated. The estimated SS is compared with the true SS in the meaning of ‘‘Data Fitting Error,’’ ε_f , which is defined by

$$\varepsilon_f = \sqrt{\frac{1}{N_b} \sum_{i=1}^{N_b} \sum_{d=1}^D \left\| \mathbf{v}_d^{(i)} - \hat{\gamma}_d^{(i)} \mathbf{a}(\hat{\vartheta}_d^{(i)}) \right\|^2} \quad (12)$$

where the superscript ‘ (i) ’ denotes the burst number, $\mathbf{a}(\hat{\vartheta}_d^{(i)})$ denotes the estimated EAMV at the i -th burst, $N_b = 652$ is the numbers of total bursts, and $\hat{\gamma}_d^{(i)}$ denotes the estimate of the fading coefficient at the i -th burst. In fact, ε_f can be regarded as a measure representing the proximity of the estimated SS with respect to the true SS. For comparison, we also construct the Array Mode Vector (AMV) by using only DOA estimates and calculate ε_f .

Next, we investigate the error contained in the EAMV given by Eq. (2) and the error due to the number of finite samples. The additional criteria for these errors are established by

$$\varepsilon_a = \sqrt{\frac{1}{N_b} \sum_{i=1}^{N_b} \sum_{d=1}^D \left\| \mathbf{v}_d^{(i)} - \gamma_d^{(i)} \mathbf{a}(\vartheta_d^{(i)}) \right\|^2}, \quad (13)$$

$$\varepsilon_n = \sqrt{\frac{1}{N_b} \sum_{i=1}^{N_b} \left\| \hat{\mathbf{E}}_S^{(i)} \hat{\mathbf{\Sigma}}^{(i)} - \mathbf{E}_S^{(i)} \mathbf{\Sigma}^{(i)} \right\|_F^2} \quad (14)$$

where $\hat{\mathbf{\Lambda}}^{(i)} = \hat{\mathbf{\Sigma}}^{(i)} \hat{\mathbf{\Sigma}}^{(i)} = \hat{\mathbf{\Lambda}}_s^{(i)} - \hat{\sigma}_n^2 \mathbf{I} \in \mathbb{R}^{D \times D}$ is a positive definite weighting matrix, $\mathbf{E}_S$ and $\mathbf{\Sigma}$ are obtained from the ensemble average covariance $\mathbf{R}$. ε_a and ε_n may be regarded as the first-order approximation error of the EAMV and the error due to finite samples,

[†]In the viewpoint of the complex angle estimation, this idea is similar to the matrix pencil method [13] for estimating complex parameters of exponentially damped sinusoids.

respectively. ε_a is denoted by “Approx. Error” to distinguish it from ε_f in figures. We represent several examples that the error of the first-order approximation dominates the one due to finite samples, i.e., $\varepsilon_a > \varepsilon_n$. Such examples are shown in Figs. 2, 5, and 6.

In figures, the AS is defined as σ_θ denoting the standard deviation of the angular deviations, $\tilde{\theta}_{dk}, k = 1, \dots, K_d$, for all d . To clarify the ratio of σ_θ over the beamwidth of array antenna, the AS normalized by a half of the 3dB beamwidth of array antenna, $\sigma_a = 2\sigma_\theta/\text{BW}_{3\text{dB}}$ [15], is also represented. σ_a helps us know the limit of the allowable AS to decide ε_f required for an application.

BPSK modulated signals are generated. At BS, the received signal related to each emitted signal is modeled as a superposition of $K_d = 32$, $d = 1, \dots, D$, i.i.d. local scattered plane waves. Each scattered wave experiences Rayleigh Fading [16]. The arrival angles for all scattered waves at BS are Gaussian distributed with the standard deviation, σ_θ . Throughout simulations, the average Signal-to-Noise Ratio (SNR) is calculated for each element. During the observation time of each burst, the SS is constant, i.e., the slow fading channel is assumed. The ULA with the interelement spacing of half-wavelength is employed. The numerical examples represent the following several points clearly.

- ε_f obtained from both the estimated AMV and the estimated EAMV.
- the variation of ε_f due to an increase in the number of array antenna elements, L .
- the relation among ε_f , ε_a , and ε_n .
- the variation of ε_f due to the number of samples, N , which is compared with the case of $\sigma_\theta = 0$.

4.1 One Signal Case

One signal with the DOA $\theta = 0^\circ$ relative to the array broadside was assumed to be received. The number of antenna elements was $L = 6$, the average SNR was

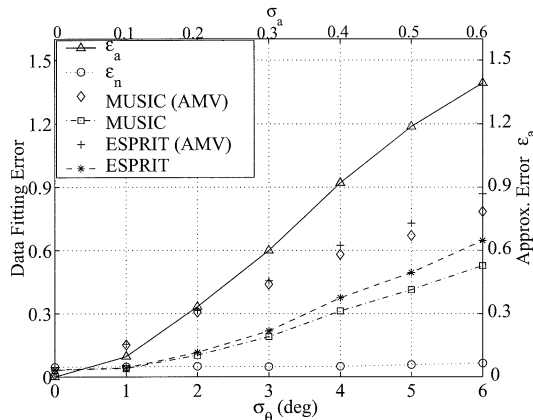


Fig. 2 Data fitting error for different σ_θ : One signal case.

14.8 [dB], and $N = 127$ samples were taken. In Fig. 2, ε_f , ε_a , and ε_n for different σ_θ are illustrated. The case that uses only the AMV estimates is defined as MUSIC (AMV) and ESPRIT (AMV), whereas the case that uses the EAMV estimates is defined as MUSIC and ESPRIT. ε_f obtained from the estimated AMV is larger than that by the estimated EAMV. ε_f obtained by using MUSIC and ESPRIT are smaller than ε_a . The difference is remarkable when σ_θ becomes large. From this, it is clear that MUSIC and ESPRIT for joint estimation can obtain more accurate SS estimates than the conventional methods considering only the DOA. This result also shows that the EAMV estimates can reconstruct the SS more accurately than the EAMV given by Eq. (2).

To help us understand the results in Fig. 2, the estimated signal subspaces spanned by the AMV and the EAMV obtained from MUSIC are conceptually and geometrically depicted in Fig. 3. In fact, the true SS $\mathbf{v}_1$ no longer belongs to the conventional array manifold over the real angle, $\{\mathbf{a}(\theta_i)\}_{\theta_i \in \mathbb{R}}$, due to the AS. $\mathbf{v}_1$ will indicate the specific point that belongs to L -dimensional complex space, as shown in Fig. 3. The conventional MUSIC finds $\mathbf{a}(\hat{\theta}_1)$ by scanning the trace of array manifold over real angle and then cannot find $\mathbf{v}_1$ accurately. On the other hand, when MUSIC for the joint estimation is used, we find resultantly the EAMV estimate closer to the true SS by searching the EAMV in the complex angle planes (see also Reference [6]). Note that the estimated EAMV is more closer to $\mathbf{v}_1$ than the EAMV given by Eq. (2). It follows that the improved accuracy in the SS estimation based on the EAMV will be one of important factors to enhance the performance of mobile communication system. It has shown [8], [9] that the estimated EAMV can increase considerably the array output SINR as compared to the case of using the estimated AMV, when MUSIC for the joint estimation is used for the signal combining.

Next, we know that the effect of ε_a becomes increasingly larger as σ_θ increases, as compared with ε_n . This means that the effect due to the finite samples acts

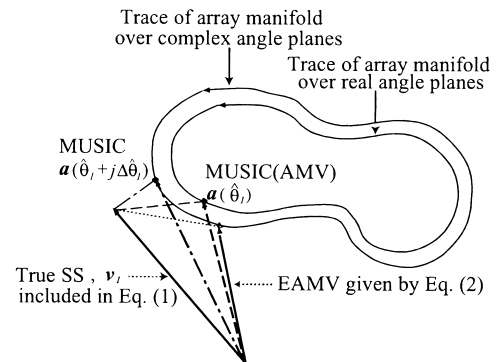


Fig. 3 Geometry of the signal subspace spanned by the EAMV estimate, which is obtained from MUSIC.

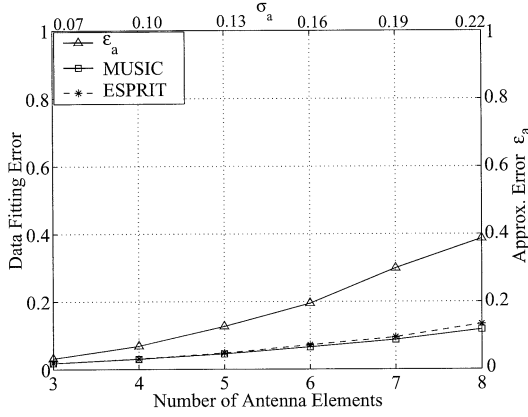


Fig. 4 Data fitting error for different antenna elements when $\sigma_\theta = 1.5^\circ$.

as a marginal factor in the AS environments. For ε_f , MUSIC shows better performance than ESPRIT. For instance, to obtain $\varepsilon_f = 0.3$ from the estimates, the allowable AS is $\sigma_\theta = 3.95^\circ$ for MUSIC, and $\sigma_\theta = 3.5^\circ$ for ESPRIT.

Figure 4 shows ε_f for different number of antenna elements when $\sigma_\theta = 1.5^\circ$. The average SNR was 15.2 [dB]. The fewer the elements, the smaller ε_f . It is natural that the error of the EAMV estimates increases as L becomes large. Practically, for the fixed σ_θ , the large-sized array antenna brings about the relative increase in σ_a . Thus, ε_f becomes larger as L increases. It also implies that the estimation accuracy of the DOA and the AS are closely related to σ_a , which gives the allowable AS with relation to the 3dB beamwidth of antenna. If we focus on the proximity between the EAMV estimate and the true SS in the AS environments, it is found that the large-size antenna itself is not always advantageous for the improvement of the estimation performance.

Finally, we know from Figs. 3 and 4 that ε_f in MUSIC is smaller than in ESPRIT. For one signal case, these results are good examples that MUSIC may provide the best fit solution, as mentioned in Sect. 3.3.

4.2 Two Signals Case

Two signals with their DOAs $\theta_1 = -10^\circ$, $\theta_2 = 10^\circ$ were received. The ULA with $L = 6$ was used and $N = 127$ samples were taken. In Fig. 5, ε_f for different σ_θ were represented. The average SNRs of the two signals were 14.9 [dB] and 15.2 [dB], respectively. As in the one signal case, ε_f in MUSIC and in ESPRIT are smaller than ε_a . However, ε_f in MUSIC is comparable to ESPRIT. Figure 5 also represents ε_n . As σ_θ increases, the effect of the finite samples on the EAMV estimates is negligible as compared with ε_a .

Figures 6(a) and (b) show ε_f for different numbers of samples when $\sigma_\theta = 0^\circ$, 1.5° , respectively. It is clear that $\varepsilon_a = 0$ when $\sigma_\theta = 0^\circ$, and ε_f decreases gradually.

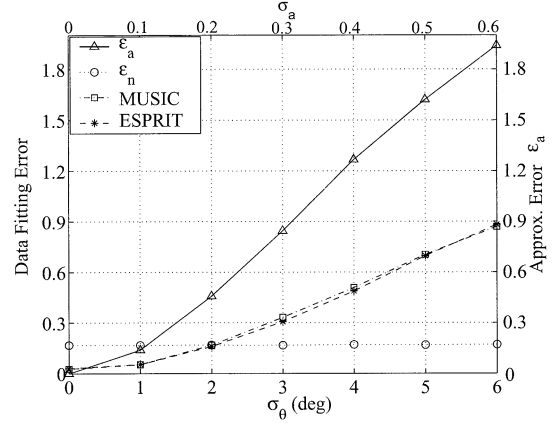
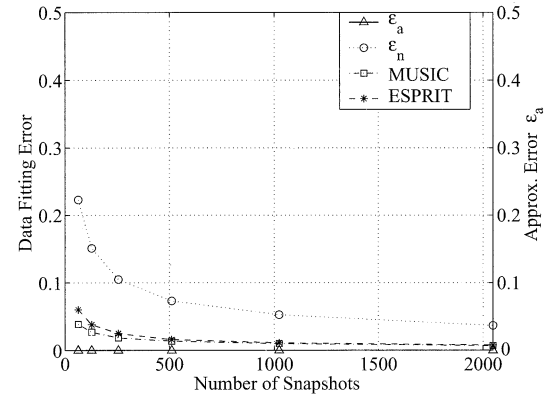
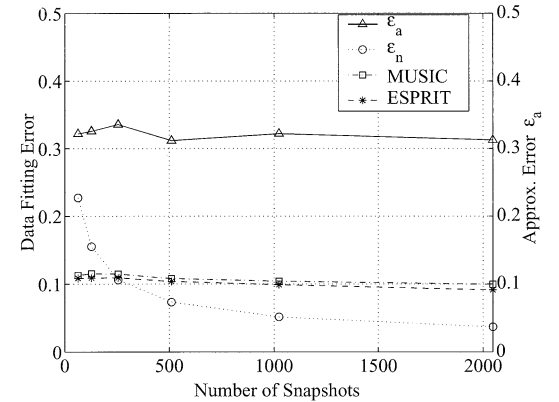


Fig. 5 Data fitting error for different σ_θ : Two signal case.



(a) Data fitting error when $\sigma_\theta = 0^\circ$.



(b) Data fitting error when $\sigma_\theta = 1.5^\circ$.

Fig. 6 Data fitting errors for different snapshots.

For this case, it was found that the variance of the DOA estimates corresponding to ε_f decreases in the order of $O(1/\sqrt{N})$. This result agrees well with the plane wave signal case, as described in [17]. However, when $\sigma_\theta = 1.5^\circ$, we know that ε_n is not the major factor deteriorating the estimation performance of the EAMV. As a result, the improvement effect of ε_f was not found even for a larger N .

5. Conclusions

In multiple scattering environments, the performance of MUSIC and ESPRIT for joint estimation of the DOA and the AS were analyzed. The criteria for performance evaluation were established and evaluated through computer simulations. It is concluded as follows:

- If we focus on the proximity between the EAMV estimates and the true SSs, the small-sized antenna may improve the estimation performance for MUSIC and ESPRIT.
- The effect due to finite samples is not dominant as compared with the plane waves model, since the first approximation error becomes dominant even for small AS.
- MUSIC and ESPRIT for joint estimation can reduce the effect of modeling error by estimating both the DOA and the AS.

For further study, the performance evaluation of the estimators with higher accuracy, such as the multidimensional MUSIC and the weighted-subspace method [14], should be done. Using these estimators, we may obtain the estimates of the SS that closely approximate the true SS. In practice, our concern is how much the SS estimates can contribute to the performance enhancement of array antenna systems.

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