Accurate Angle-of-Arrival Estimation Method in Real System by Applying Calibration and Spatial Smoothing

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SUMMARY Although subspace-based methods for estimating the Angle of Arrival (AOA) require a precise array response to achieve highly accurate results, it is difficult to obtain this response in practice even though the antennas are calibrated. Therefore, a method of compensating for errors in calibration is required. This paper proposes a procedure to enable precise AOA estimates to be obtained in a real system by applying array calibration and spatial smoothing preprocessing (SSP). Measured data were collected from experiments using two scenarios, i.e., in an anechoic chamber and at an open site, where a single source signal arrived at the array antenna. All measured data were then calibrated by using data obtained at 0 deg in an anechoic chamber before the AOs were estimated. Nevertheless, errors in the array response remained after calibration because errors in the AOA estimates could still be observed. SSP was then applied to the calibrated data to obtain more accurate AOA estimates. We found that SSP can reduce the random error in an array response obtained in a real system, leading to reduced errors in AOA estimates in the observed data. To generalize the problem that SSP can reduce random perturbation in the array response, simple expressions are illustrated and verified by Monte-Carlo simulation. Random gain and phase errors in the array response are only considered in this paper and ESPRIT was used to estimate the AOs.

key words: AOA estimation, calibration, spatial smoothing

1. Introduction

The angle of arrival (AOA) information of an impinging signal on an array antenna can be utilized for many applications such as smart antennas and mobile localization systems. Especially, for mobile localization systems, precise AOA estimation is required to know the accurate position of the mobile terminal. The desired accuracy of the localization system is dictated by the application requirements and the ambient conditions. For example, the emergency 911 (E-911) service outlined in the 1996 ruling issued by the U.S. Federal Communications Commission (FCC) requires wireless carriers to report the location of mobile terminals with an accuracy of 125 m in 67 percent of the cases [1]–[2]. The AOA accuracy depends not only on the location accuracy requirement, but also on the distance between the transmitter and receiver. For example, to achieve a ±10 m ground location accuracy from a radiolocation system carried by a stratospheric aircraft flying at 20 km altitude, a very high AOA accuracy of less than 0.03 deg is needed [3].

A variety of AOA-estimation algorithms have been extensively discussed in the literature [4] over the past few decades, especially subspace-based methods because of their high-resolution capabilities. Well known subspace-based algorithms are the multiple signal classification (MUSIC) algorithm [5] and the estimation of signal parameters via rotational invariance techniques (ESPRIT) [6]. These subspace-based approaches are only highly accurate when an exact array response is obtained. Therefore, their accuracy in computer simulation is outstanding, while this becomes degraded in actual applications because of difficulties with obtaining the exact array response. Deviations from the exact array response due to mutual coupling, gain and phase mismatch between the array elements, and element-position errors can seriously degrade the performance of high-resolution algorithms. Therefore, to achieve the exact array response, an array calibration is necessary and has been extensively researched. Calibration methods that deal with the effects of mutual coupling, mismatch in array elements, and element-positioning error have been proposed [7]–[10]. An interpolation procedure to reduce the number of points required to calibrate arrays has also been proposed [11]. Further, there has been some research on techniques of calibration by applying data obtained from actual scenarios [12]–[15]. Nevertheless, it has been found that the array calibration cannot entirely remove errors in the array response, thus leading to errors in estimated AOs. The authors have also proposed calibration techniques by applying the data obtained from real scenarios with the measurement system [16] and found that although slightly improved AOA estimates could be obtained, calibration errors still remained in the array response. An investigation on the imperfection of the array calibration based on measured data obtained with the measurement system has also been done [17]. To achieve more precise AOA estimates, a method of compensating for calibration errors in an array response is needed. The performance analysis of subspace-based methods due to errors perturbed in the array response have been presented [18], [19]. Further studies on the effects of spatial smoothing preprocessing (SSP) on the performance of subspace-based methods with array model errors have also been done [20], [21]. Their results revealed that SSP can improve the performance of subspace methods, especially ESPRIT and the Minimum-Norm method, when there is perturbation in the exact array response. Nevertheless, although theoretical expressions for the Mean Squared Error (MSE) in AOA were well derived, they could only be validated by simulation.
We investigated the possibility of using SSP for reducing the random error in an array response in an actual system by using a measurement system that we had reported earlier [22]. The experiments were conducted using two scenarios to obtain two sets of measured data: (1) in an anechoic chamber and (2) at an open site. The measured data obtained in the anechoic chamber at 0 deg were used for calibrating all AOAs obtained in both the anechoic chamber and at the open site by utilizing the amplitude and phase compensation technique [16], [17]. The results revealed that the accuracy of AOA estimates obtained from measured data could be improved by applying SSP to ESPRIT. These results agreed with previously presented simulation results [20], [21].

This paper discusses an extension of our recent work [22] and the generalized expressions with gain and phase errors perturbed in the array response are explained in more detail. In addition, the optimal number of sub-arrays is investigated to obtain the most effective SSP. This paper is organized as follows. Section 2 briefly explains the procedure for processing data starting with a description of an ideal array model, AOA estimates using ESPRIT, and SSP. The capability for reducing random error in an array response using SSP is demonstrated by simple formulas and our investigation into the optimal number of sub-arrays is discussed in Sect. 3 and the simulation results are discussed in Sect. 4. Section 5 describes the measurement system and the conditions under which the experiments in the anechoic chamber and at the open site were conducted. Section 6 describes and discusses the experimental results. Finally, the conclusion is summarized in Sect. 7.

2. Algorithm for Estimating AOAs

2.1 Signal Model

By considering a single narrowband signal impinging on an \( M \)-element uniform linear array (ULA), an array output vector, \( \mathbf{x}(t) \), can be modeled as

\[
\mathbf{x}(t) = \mathbf{a}s(t) + \mathbf{n}(t),
\]

where \( s(t) \) is an arriving signal at time \( t \), \( \mathbf{n}(t) \) is an additive white Gaussian noise vector, and \( \mathbf{a} \) is a so-called steering vector describing the array response for a source with an arrival angle, \( \theta \), which is assumed to be known from calibration procedures. The ideal steering vector can be defined as

\[
\mathbf{a}(\theta) = [1, e^{j\omega}, e^{j2\omega}, \ldots, e^{j(M-1)\omega}]^T,
\]

where \( \omega = 2\pi d \sin \theta / \lambda \) is the phase difference between adjacent elements, \( d \) is the inter-element spacing, \( \lambda \) is the signal wavelength, and the superscript \( T \) denotes the transpose.

The output covariance matrix of the single source can be written as

\[
\mathbf{R} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)]=\sigma^2_\mathbf{s}\mathbf{a}\mathbf{a}^H + \sigma^2_\mathbf{n}\mathbf{I},
\]

where \( \mathbb{E}[\cdot] \) represents the statistical expectation, the superscript \( H \) denotes the complex conjugate transpose operator, \( \sigma^2_\mathbf{s} \) is the arriving-signal power, and \( \sigma^2_\mathbf{n} \) is the noise covariance matrix that reflects the uncorrelated noise among all elements, having a common variance at all elements.

For its eigendecomposition, the output covariance matrix can be written as

\[
\mathbf{R} = \lambda_i \mathbf{u}_i \mathbf{u}_i^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H,
\]

where \( \mathbf{u}_i \) is an eigenvector and \( \lambda_i \) is an eigenvalue forming the signal subspace. \( \mathbf{U}_n \) represents the matrix containing noise eigenvectors written as

\[
\mathbf{U}_n = [\mathbf{u}_2, \ldots, \mathbf{u}_M],
\]

and \( \Lambda_n \) is a diagonal matrix with diagonal elements \( \lambda_2 = \ldots = \lambda_M = \sigma^2_\mathbf{n} \). For estimating AOA, the ESPRIT algorithm, which uses the structure of ULA steering vectors, is utilized. The AOA can be obtained by determining the eigenvalues of the rotation vector (\( \Psi \)) given by

\[
\mathbf{u}_2 = \mathbf{u}_1 \Psi,
\]

using the methods of least squares (LS), or total least squares (TLS) [6] where \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) are the signal subspace vectors by respectively removing its last and first rows.

2.2 Spatial Smoothing Preprocessing (SSP)

SSP was proposed to de-correlate coherent sources present in the scene. However, our purpose in using SSP here was to investigate if it could reduce the errors in the array calibration and thereby improve the accuracy of AOA estimates. This technique is a preprocessing scheme to partition the total array of elements into sub-arrays and then generate the average for the sub-array output covariance matrices. In forward-only spatial smoothing (FSS) [23], [24], an array is divided into \( L \) smaller sub-arrays, each of which contains \( M_s \) elements (see Fig. 1), where \( L = M - M_s + 1 \). Let \( \mathbf{x}_l(t) \) stand for the output vector of the \( l \)-th sub-array; then,\n
\[
\mathbf{x}_l(t) = [x_{1l}(t), x_{2l}(t), \ldots, x_{M_s l}(t)]^T
\]

where \( \mathbf{a}_l = [a_{l1}, a_{l2}, \ldots, a_{lM_s l-1}]^T \), and \( \mathbf{n}_l(t) = [n_{1l}(t), n_{2l}(t), \ldots, n_{M_s l}(t)]^T \).

Then, the covariance matrix of the \( l \)-th forward sub-array is obtained by

\[
\mathbf{R}_l = \mathbb{E}[\mathbf{x}_l(t)\mathbf{x}_l^H(t)] = \sigma^2_\mathbf{s}\mathbf{a}_l\mathbf{a}_l^H + \sigma^2_\mathbf{n}\mathbf{I}.
\]

Following Shan et al. [23], the FSS covariance matrix, \( \mathbf{R}_F \), is defined as the average value for the covariance matrices of all forward sub-arrays:

\[
\mathbf{R}_F = \frac{1}{L} \sum_{l=1}^{L} \mathbf{R}_l.
\]

![Fig. 1](image)

The forward/backward spatial-smoothing scheme.
The smoothed covariance matrix, $\mathbf{R}_F$, is used instead of the covariance matrix, $\mathbf{R}$, to estimate AOA.

### 3. Reduced Random Error in Array Response Using SSP

#### 3.1 Error in Array Model

The modeling of error in the array response is discussed in this subsection.

Let $a_m(\theta) = e^{j\phi_m}$ be the nominal array response of the $m$th element and $\phi_m = (m - 1)\omega$. When amplitude and phase perturbations in the array antenna occur, the perturbed array response of the $m$th element can be modeled [18] as

$$a_m(\theta) = (1 + \bar{g}_m) e^{j(\phi_m + \tilde{\phi}_m)} = \gamma_m a_m(\theta),$$

(10)

where $\gamma_m = (1 + \bar{g}_m)e^{j\tilde{\phi}_m}$, and $\bar{g}_m$ and $\tilde{\phi}_m$ are the respective gain and phase errors of the $m$th element, which are assumed to be very small and independent of the AOA. Then, using the first order of Taylor expansion, $\gamma_m$ can be approximated as

$$\gamma_m \approx 1 + \bar{g}_m + j\tilde{\phi}_m = 1 + \xi_m,$$

(11)

where $\xi_m = \bar{g}_m + j\tilde{\phi}_m$ denotes a zero-mean complex Gaussian error of the $m$th element with known covariance. As assuming that the gain and phase perturbations of each element were independent identically distributed (i.i.d.) Gaussian random variables with respective variances $\sigma_g^2$ and $\sigma_\phi^2$, the covariance of the random error in each element can be obtained by

$$E[\xi_m\xi_m^*] = \sigma_g^2 + \sigma_\phi^2.$$

(12)

Variances $\sigma_g^2$ and $\sigma_\phi^2$ determine the deviation in the gain and phase response from their respective nominal values. Generally, deviation in gain is defined in decibels as $20 \log_{10}(\sigma_g)$, and deviation in phase can be defined as $180(\sigma_\phi/\pi)$ deg.

The perturbed array output vector, $\mathbf{x}(t)$, can be modeled from the above description as

$$\mathbf{x}(t) = \Gamma \mathbf{a} s(t) + \mathbf{n}(t) = \tilde{\mathbf{a}} s(t) + \mathbf{n}(t),$$

(13)

where $\Gamma = \text{diag}[\gamma_1, \gamma_2, \ldots, \gamma_M]$; then, the perturbed output covariance matrix, $\tilde{\mathbf{R}}$, can be expressed as

$$\tilde{\mathbf{R}} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \sigma_n^2 \tilde{\mathbf{a}} \tilde{\mathbf{a}}^H + \sigma_n^2 \mathbf{I}.$$

(14)

#### 3.2 Capability of SSP for Reducing Random Error in Array Response

Since Hari’s work [21] has already reported that, for the single source case, the theoretical expressions for the Mean Square Error (MSE) in AOA for ESPRIT with FSS and FBSS are same, only FSS preprocessing has been considered. According to (9), the perturbed forward spatially smoothed covariance matrix, $\tilde{\mathbf{R}}_F$, is obtained by

$$\tilde{\mathbf{R}}_F = \frac{1}{L} \sum_{l=1}^{L} \tilde{\mathbf{R}}_l^f,$$

$$= \sigma_n^2 \frac{1}{L} \sum_{l=1}^{L} \tilde{\mathbf{a}}_l \tilde{\mathbf{a}}_l^H + \sigma_n^2 \mathbf{I},$$

(15)

where $\tilde{\mathbf{R}}_l^f$ is the perturbed covariance matrix of the $l$th forward sub-array and defined as

$$\tilde{\mathbf{R}}_l^f = E[\mathbf{x}_l^f(t)\mathbf{x}_l^f(t)^H] = \sum_{j=1}^{L} \tilde{\mathbf{A}}_l^f \tilde{\mathbf{A}}_l^f \mathbf{I}.$$

(16)

and $\mathbf{x}_l^f(t)$ is the perturbed output vector of the $l$th forward sub-array

$$\mathbf{x}_l^f(t) = [\mathbf{x}_l(t), \mathbf{x}_{l+1}(t), \ldots, \mathbf{x}_{l+L-1}(t)]^T.$$  

(17)

Since gain and phase errors are independent of each other, their analyses can be separately discussed.

#### 3.3 Phase Error in Array Response

By assuming that only the phase response of the array elements deviates from the nominal response, the perturbed steering vector of the $l$th forward subarray $\tilde{\mathbf{a}}_l$ can be written as

$$\tilde{\mathbf{a}}_l = \begin{bmatrix} (1 + j\tilde{\phi}_1) e^{j\theta_1} \\ (1 + j\tilde{\phi}_2) e^{j\theta_2} \\ \vdots \\ (1 + j\tilde{\phi}_{L-1}) e^{j\theta_{L-1}} \end{bmatrix}.$$  

(18)

Let us only focus on part of the array response in (15), which is expanded in (19) at the top of next page. The matrix $\mathbf{B}$ is obtained by averaging all matrices $\mathbf{B}_l$ over $L$ where $\mathbf{B}_l = \tilde{\mathbf{a}}_l \tilde{\mathbf{a}}_l^H$. Each element of matrix $\mathbf{B}_l$, $b_{pq,l}$, consists of two parts: i) part of the phase shift of the nominal array response in terms of $e^{j\phi_{p-q} \omega}$, which is constant for each matrix $\mathbf{B}_l$ and ii) part of the random error in terms of $(1 + j(\tilde{\phi}_{l+p-1} - \tilde{\phi}_{l+q-1}))$, where $p$ denotes the row of the matrix and $q$ denotes its column.

By considering only the lower diagonal part of matrix, $b_{pq,l}$ can be written as

$$b_{pq,l} = (1 + j(\tilde{\phi}_{l+p-1} - \tilde{\phi}_{l+q-1})) e^{j\phi_{p-q} \omega},$$

(20)

where $p > q$. First, focusing on the first column, $(q = 1)$, the element for the $p$th-row and first-column of matrix $\mathbf{B}$, $b_{p1}$, can be written as
\[ B = \frac{1}{L} \sum_{l=1}^{L} B_l \]
\[ = \frac{1}{L} \left[ \sum_{l=1}^{L} \left( 1 + j(\hat{\phi}_l - \hat{\phi}_l) e^{j\omega} \right) \right] \]
\[ = \frac{1}{L} \left[ \sum_{l=1}^{L} \left( 1 + j(\hat{\phi}_l + M_0 - \hat{\phi}_l) e^{j(M_0-1)\omega} \right) \right] \]
\[ = \frac{1}{L} \left[ \sum_{l=1}^{L} \left( 1 + j(\hat{\phi}_l - \hat{\phi}_l) e^{j\omega} \right) \right] \]
\[ = \frac{1}{L} \left[ \sum_{l=1}^{L} \left( 1 + j(\hat{\phi}_l + M_0 - \hat{\phi}_l) e^{j(M_0-2)\omega} \right) \right] \] (19)

\[ b_{p1} = \frac{1}{L} \sum_{l=1}^{L} b_{p1,l} \]
\[ = \frac{1}{L} \left[ \sum_{l=1}^{L} (1 + j(\hat{\phi}_l - \hat{\phi}_l) e^{j\omega}) \right] \]
\[ = e^{j(p-1)\omega} \left[ 1 + \frac{j}{L} \left( \sum_{l=1}^{L} \phi_l - \sum_{l=1}^{L} \phi_l \right) \right] \]
\[ = e^{j(p-1)\omega} \left[ 1 + j\zeta_{p1} \right] \] (21)

As previously assumed that the phase errors of each element were i.i.d. normal random variables with zero mean, i.e.,
\[ \tilde{\phi} \sim N\left(0, \sigma^2_\phi\right), \] (22)
and then referring to (21), a new random phase, \( \zeta_{p1} \), is distributed with
\[ \zeta_{p1} \sim N\left(0, 2(p - 1) \sigma^2_\phi / L^2 \right). \] (23)

In the same manner as the arbitrary column, \( q \), a random phase, \( \zeta_{pq} \), is distributed with
\[ \zeta_{pq} \sim N\left(0, 2 \mid p - q \mid \sigma^2_\phi / L^2 \right) \] for \( p \neq q \). (24)

It can easily be observed that the deviation of phase errors decreases according to the number of sub-arrays, \( L \). However, there is a trade-off. If the number of sub-arrays is large (i.e., an array size, \( M_0 \), is small), the potential resolution becomes poor due to the small working aperture. Therefore, the optimal number of sub-arrays needs to be investigated to obtain the most effective SSP. The optimal number of sub-arrays in the case of phase error has already been reported \([21]\). By using ESPRIT to estimate AOA, this can be obtained by
\[ L_{\text{opt}} = \frac{2M}{3} \] (26)
for \( L > M_0 \).

3.4 Gain Error in Array Response

By assuming that only the gain response of the array elements deviates from the nominal response, the perturbed steering vector of the \( \ell \)th forward sub-array, \( \tilde{a}_\ell \), can be written as
\[ \tilde{a}_\ell = \begin{bmatrix} (1 + \tilde{g}_\ell) e^{j\phi_\ell} \\ (1 + \tilde{g}_{\ell+1}) e^{j\phi_{\ell+1}} \\ \vdots \\ (1 + \tilde{g}_{\ell+M_0-1}) e^{j\phi_{\ell+M_0-1}} \end{bmatrix}. \] (27)

Analyzing the gain error in the same manner as the phase error, \( b_{pq,j} \) can be obtained by
\[ b_{pq,j} = (1 + \tilde{g}_{\ell+p-1} - \tilde{g}_{\ell+q-1}) e^{j(p-q)\omega}. \] (28)

Then, the element for the arbitrary \( p \)th-row and \( q \)th-column of matrix \( B \), \( b_{pq} \), can be written as
\[ b_{pq} = \frac{1}{L} \sum_{l=1}^{L} b_{pq,l} \]
\[ = e^{j(p-q)\omega} \left[ 1 + \frac{1}{L} \sum_{l=1}^{L} (\tilde{g}_{\ell+p-1} + \tilde{g}_{\ell+q-1}) \right] \]
\[ = e^{j(p-q)\omega} \left[ 1 + \eta_{pq} \right]. \] (29)

Also, as the authors previously assumed that the gain errors of each element were i.i.d. normal random variables with zero mean, i.e.,
\[ \tilde{g} \sim N\left(0, \sigma^2_g\right), \] (30)
then, referring to (29), a new random gain, \( \eta_{pq} \), is distributed with
\[ \eta_{pq} \sim N\left(0, 2\sigma^2_g / L \right). \] (31)
It is seen that the deviation of gain errors also decreases according to the number of sub-arrays, \( L \), due to SSP. Also, the optimal number of sub-arrays in the case of gain error is required to provide the best performance of SSP, and is demonstrated here.

By referring to (31) when gain error is perturbed in the array response, it can be inferred that the deviation in AOA estimates could be decreased with the increased number of sub-arrays, \( L \). It is also well known that high resolution can be achieved with a large number of array elements. Therefore, AOA estimation errors caused by gain perturbation by applying SSP, denoted by \( \varepsilon_g \), is inversely proportional to the number of sub-arrays, \( L \), and the number of sub-array elements, \( M_0 \),

\[
\varepsilon_g \propto \frac{1}{LM_0},
\]  

(32)

where \( M_0 = M - L + 1 \). Then, (32) can be rewritten as

\[
\varepsilon_g \propto \frac{1}{L(M - L + 1)}.
\]  

(33)

The optimal number of sub-arrays can be determined by maximizing the denominator of (33). Therefore, by taking its first derivative, the optimal number of sub-arrays in the case of gain error can be obtained by

\[
L_{\text{opt}} = \frac{M + 1}{2}.
\]  

(34)

4. Simulation Results and Discussion

Computer simulations were conducted to demonstrate how well the proposed method performed. The ULA with 8 elements (\( M = 8 \)) and half a wavelength of element spacing was used. A signal with an AOA of 6 deg was considered with 500 snapshots and 100 Monte-Carlo trials were performed for each simulation.

Figure 2 compares the standard deviation (STD) of AOA estimates versus the number of sub-arrays (\( L \)) with phase error of \( \approx 3 \) deg (0.05 radian) for simulation and theoretical expression proposed by [21]. The simulation result agrees well with the evaluation of the theoretical expression. As previously mentioned, the expressions of MSEs in AOA for ESPRIT with FSS and FBSS for the one source case are the same [21], therefore, in all simulations, only the results of FSS are shown.

Figure 3 shows the STD of AOA estimates versus the number of sub-arrays (\( L \)) with different SNRs when a phase error (\( \sigma_{\phi} \)) of \( \approx 3 \) deg (0.05 radian) is perturbed in the array response. It is seen that the results for AOA estimates have not been improved with higher SNRs. Nevertheless, an optimal number of sub-arrays can be observed at 3 and 5, which are respectively in accordance with (25) and (26), yielding the best results for AOA estimates.

Figure 4 depicts the STD of AOA estimates versus the varied random-phase errors. AOA is estimated by the ESPRIT algorithm with and without SSP. The authors used FSS with different sub-arrays of \( L = 2, 3, 4, 5 \) and 6. The SNR was 10 dB in this simulation. It is seen that the STD of AOA estimates by applying FSS with the number of sub-arrays shown is smaller than that without applying FSS.
Figure 5 plots the STD of AOA estimates versus the number of sub-arrays ($L$) with different SNRs when the gain error ($\sigma_g$) of $-10$ dB is perturbed in the array response. Different to the case of phase error, the results indicate that the AOA estimates have smaller deviations with higher SNR. Moreover, an optimal sub-array of 4, which verifies (34), can be seen to obtain the most effective SSP even if SNR is low. Therefore, it is possible to avoid gain perturbation in array responses without applying SSP when SNR is high, or when an optimal number of sub-arrays is used for SSP even with low SNR.

The STD of AOA estimates versus the varied random-gain errors are plotted in Fig. 6. AOA is also estimated by the ESPRIT algorithm with and without SSP. In this simulation, different numbers of sub-arrays for FSS were used, i.e., $L = 2, 3, 4, 5$ and 6 and the SNR was $10$ dB. It is also seen that the STD of AOA estimates by applying FSS with the number of sub-arrays shown is smaller than that without applying FSS.

5. Measurement System

Measured data were needed for processing to demonstrate that this method could be applied in practice. Therefore, the experiments were conducted using two scenarios: (1) in an anechoic chamber to obtain the data with no scattering effect but influenced by a phase curvature of the wavefront; and (2) at an open site under LOS condition to obtain the data with a plane wavefront but possibly affected by scatterers. The same system of measurement was used for both scenarios, except for the transmitting antenna.

5.1 Equipment for Experiment

The same set of equipment was used on the receiver side for both experimental scenarios. The receiving antenna is depicted in Fig. 7 and its specifications are listed in Table 1. The receiving antenna was composed of two parallel sets of ULAs and the authors used “Array 1” to refer to the upper ULA and “Array 2” to refer to the lower ULA. The spacing between the two sets of array antennas was $35$ cm (measured center-to-center).

The data received from the array antenna were down-converted to an intermediate frequency (IF) of $1.9648$ MHz, then digitized with a sampling rate of $4.375$ MHz, and further downconverted to the baseband. Then this baseband data were used for the AOA estimates. There is a block diagram of the receiver equipment in Fig. 8. Only the eight middle elements in each ULA were used since the receiver equipment only had 16 cable connectors, with the end elements used as dummy elements. The transmitting antennas used were a horn antenna with a $14.67$-dBi gain in the anechoic chamber and a patch antenna with a $7$-dBi gain and right-handed circular polarization (RHCP) at the open site. The frequency used was $1.74$ GHz and the received SNR

![Fig. 5](image-url) STD of AOA estimates vs. number of sub-arrays with different SNRs with gain error of $-10$ dB.

![Fig. 6](image-url) STD of AOA estimates vs. gain errors.

![Fig. 7](image-url) Receiving array antenna.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Specifications for receiving antenna.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna arrays</td>
<td>Two parallel ULAs</td>
</tr>
<tr>
<td>Number of elements ($M$)</td>
<td>8 active elements and 2 dummy elements</td>
</tr>
<tr>
<td>Element spacing ($d$)</td>
<td>$0.81$ ($\approx 14$ cm)</td>
</tr>
<tr>
<td>Antenna type</td>
<td>Patch</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>$7$-dBi</td>
</tr>
<tr>
<td>Polarization</td>
<td>RHCP</td>
</tr>
<tr>
<td>Size of array</td>
<td>$135 \times 30$ cm (length$\times$height)</td>
</tr>
</tbody>
</table>
was 36 dB in both experimental scenarios.

5.2 Measurement Setup in Anechoic Chamber

The measurement setup for the experiment in the anechoic chamber is illustrated in Fig. 9. The distance between the transmitter and receiver was 10.42 m where the wavefront of the signal was perceptively curved because of near-field measurement. Therefore, the phase curvature of the near-field wavefront was compensated to a far-field wavefront before data were processed [25]. The receiving antenna was rotated in an azimuth direction adjusted by an antenna positioner controller. The experiments were repeated to obtain data within an AOA range of −6 to 6 deg.

The measured array responses of each element are shown in Figs. 10 and 11, with normalized amplitude and phase, respectively. The amplitude response of each element was normalized by the maximum value among all the elements, which was the amplitude response of 6 deg of the 8th element, while the phase response of each element was normalized by that of 0 deg.

5.3 Measurement Setup at Open Site

The field test was undertaken in an open space to identify issues arising within the realistic scenario. Moreover, since the distance between the transmitter and receiver can be adjusted to be longer than that in the anechoic chamber, a smaller array manifold error caused by distance can be attained. For example, for the outermost element with the signal arriving at 0 deg at the array antenna, a high phase error of approximately 23.3 deg was obtained when the measurement was done in the anechoic chamber with a 10.42 m distance between the transmitter and receiver. On the other hand, for the open site measurement with a longer distance between the transmitter and receiver, for example a 400 m distance, a much smaller phase error of approximately 0.6 deg was obtained. Therefore, the longer the distance between the transmitter and receiver, the smaller the error in the array manifold.

The data were obtained from a field experiment conducted in Hokkaido, Japan as shown in Fig. 12. The antenna array was mounted at a height of 10 m above the ground on top of a building. The transmitting antenna was a 1-m-high single patch antenna placed at a known position in an open area under LOS condition. The transmitting antenna was tilted upward at an angle of about 45 deg to avoid the signals reflecting from the ground. The distance between the transmitting and receiving antennas was approximately 800 m when the AOA was 0 deg. The experimental conditions are outlined in Fig. 13. The other measurement equip-
ment was the same as that used for the experiment in the anechoic chamber. The experiments were repeated 15 times by changing the position of the transmitter whose known angles of arrival relative to the receiving array antenna were $-6, -5, -2, -1, -0.5, -0.2, -0.1, 0, 0.1, 0.2, 0.5, 1, 2, 5$ and $6$ deg.

6. Experimental Results and Discussion

In each experiment, one transmitter was used, therefore, only one signal is expected to be received if no multipath effect exists. To investigate whether the multipath components arrive at the array antenna, the eigenvalue distribution of the covariance matrix needs to be shown. Figure 14 illustrates the eigenvalue distribution of the FSS covariance matrices with five sub-arrays for both the measured data in the anechoic chamber and at the open site when the signal arrives at $0$ deg at Array 1. This figure clearly shows that the FSS covariance matrices have only one dominant eigenvalue, therefore it can be inferred that no multipath effect exists in both experimental scenarios.

6.1 Results from Anechoic Chamber Data

The measured data were calibrated before AOAs were estimated. The data obtained in the anechoic chamber at $0$ deg were used to calibrate all AOAs by using the amplitude and phase compensation technique [16], [17]. Although the receiving array antenna was composed of two ULAs, as shown in Fig. 7, the data received at each ULA were processed separately. The STD of AOA estimation errors for all measured data from both Arrays 1 and 2 versus the number of sub-arrays is plotted in Fig. 15. The optimal number of sub-arrays for both Arrays 1 and 2 can be observed at 3 and 5. Therefore, either three or five sub-arrays can be chosen. The authors have compared the results by using $L = 3$ and $L = 5$ and found almost the same results. Moreover, because of the page limitation, five sub-arrays ($L = 5$) were used to estimate AOAs for all measured data. Figures 16 and 17 plot AOA estimation errors versus the exact angle for the measured data obtained from Arrays 1 and 2. It can be seen that
SSP can significantly improve AOA estimates.

6.2 Results from Open Site Data

The same data obtained in the anechoic chamber at 0 deg were applied to calibrate the open-site data before AOAs were estimated. Figures 18 and 19 plot AOA estimation errors versus the exact angle for the measured data obtained from Arrays 1 and 2. The results with and without FSS are compared.

Since the STD of AOA estimation errors for all measured data from both Arrays 1 and 2 versus the number of sub-arrays had the same trend as that for the anechoic-chamber data, the results are not shown here. However, the optimal number of sub-arrays was also observed at 3 and 5. Therefore, either three or five sub-arrays can be chosen and will give the same results of estimation. Here, five sub-arrays ($L = 5$) were used to estimate AOAs. AOA estimation errors were demonstrated to be relatively high compared to the results obtained from anechoic-chamber data. Such errors are possibly due to the non-ideal surroundings for the array antenna in the actual installation. Regardless of these limitations, the measured data are still useful to investigate the accuracy of AOA estimates based on our procedure. According to the results, more precise AOA estimates were observed when SSP was applied.

The STD of AOA estimation errors using ESPRIT and FSS-ESPRIT are listed in Table 2. It is found that a reduction in the STD of up to 0.086 deg (at maximum) can be achieved for data obtained from Array 1 at the open site when SSP was applied. This reduction has advantages especially when this system is used for mobile-localization applications because a target terminal can be detected more precisely. In our experiment with the 800 m distance between the transmitter and receiver, this reduction of 0.086 deg in the FSS-ESPRIT is equivalent to a reduction in distance error of approximately 1.2 m. This means that a target terminal can be detected with distance errors of 0.8 m compared to ESPRIT, which has a distance error of 2 m.

7. Conclusion

This paper proposed a method of precisely estimating AOAs in a real system using array calibration and spatial smoothing. Simpler expressions were presented by considering the random gain and phase errors separately to generalize problem formulas that SSP can reduce random errors in an array response. The improvement in performance was verified by Monte-Carlo simulation. The optimal number of sub-arrays was also investigated to find the most effective SSP. According to the simulation results, it is also found that the gain perturbation in the array response could be avoided with a high SNR while this was not the case for phase perturbation.
To investigate that our proposed technique is applicable for real systems, experiments were conducted in an anechoic chamber and at an open site. The measured data were calibrated before the AOAs were estimated, then SSP was applied to compensate for errors in calibration. The results revealed that SSP improves the accuracy of AOA estimates using ESPRIT. Therefore, this approach is applicable to real scenarios, at least to the extent verified by our measurement system and under the field experiment conditions. Moreover, it is found that the optimal number of sub-arrays observed from results for the STD of AOA estimation error for the measured data was in good agreement with theoretical expressions for the case of phase error.

Consequently, our measured data were possibly only affected by phase errors in the array response because of the two above observations: i) the relatively high received SNR used and ii) agreement of the optimal number of sub-arrays with theoretical expressions.

References


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