Signal Subspace Interpolation from Discrete Measurement Samples in Constructing a Database for Location Fingerprint Technique

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SUMMARY In this paper, a method of the signal subspace interpolation to constructing a continuous fingerprint database for radio localization is proposed. When using the fingerprint technique, enhancing the accuracy of location estimation requires very fine spatial resolution of the database, which entails much time in collecting the data to build up the database. Interpolated signal subspace is presented to achieve a fine spatial resolution of the fingerprint database. The angle of arrival (AOA) and the measured signal subspace at known locations are needed to obtain the interpolated signal subspaces. The effectiveness of this method is verified by an outdoor experiment and the estimated location using this method was compared with those using the geometrically calculated fingerprint and the measured signal subspace fingerprint techniques.

key words: localization, fingerprint technique, array antenna, signal subspace

1. Introduction

Mobile localization has been extensively researched because of its importance to many wireless applications, e.g. [1]. Localization techniques can be classified into several categories, such as received signal strength (RSS), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), location fingerprint based techniques, and their combinations [1]–[4]. The triangulation-based localization techniques, i.e. use of AOA, TOA and TDOA need multiple base stations (BSs) to estimate the location of mobile terminals. Moreover, to obtain high resolution TOA and TDOA, a special time synchronized equipment is required. To make a localization system simpler, the fingerprint algorithm can be considered since it needs only one BS and relatively simple equipment can be utilized. In this work, the authors focus on a localization system using an array antenna as the receiver on BS and the location fingerprint technique is used to estimate the transmitter location. The basic principle of this technique is to find the location of the mobile terminal by comparing its signal pattern received by the BS with a previously recorded database of known signal-location information. With this principle, various pattern matching algorithms have been reported in the literature [5]–[7]. Although there are many researches on signal pattern matching algorithms, the advantages of the spatial information were not effectively employed. For this reason, the subspace matching technique using the received signal subspace associated with the spatial information was proposed [8], [9]. Nevertheless, by using the fingerprint techniques, the database of the signal pattern needs to be constructed in advance wherein the finer the spatial resolution of the database, the more accurate the estimate of the mobile terminal locations. To obtain a fine spatial resolution of the database, the experiment has to be repeated many times or the detailed scenario information needs to be known when using the simulation tools. This is not practical in real applications due to heavy load. Moreover, when the environment conditions change, a new database needs to be reconstructed. For feasibility in real applications, fewer data observations in making the fine database is more preferable. The authors reported the approach in constructing a fine database using some measurement data points by interpolating measured signal subspaces [10]. In that report, the signal subspace matching algorithm [8], [9] was used to estimate the transmitter location. Its performance was verified by an outdoor experiment. However, the data obtained from that experiment had a coarse spatial sampling interval of 100 m used for interpolation leading to big errors in location estimation. This paper discusses an extension of that work and the way to make a continuous fingerprint database are explained in more detail. The performance of this method is validated using other experiments of which the measured data have a finer spatial sampling interval (10 m) for interpolation. Moreover, the estimated location results are compared with those using geometrically calculated fingerprint database and the measured signal subspace fingerprint database as well as also compared with our previous work [10].

This paper is organized as follows. Section 2 explains the signal model used. Section 3 briefly reviews the location fingerprint technique and explains two straightforward approaches to constructing fingerprint database which are utilized to compare their performances with the proposed method. The proposed method in making a continuous fingerprint database using the interpolation of measured signal subspaces is explained in Sect. 4. Section 5 describes the experiment system and setup. Section 6 shows the results and gives discussions. Finally, the conclusion is given in Sect. 7.
2. Signal Model

In this work, the fingerprint database which is parameterized in the one-dimensional space is considered and is obtained through the measurements. The location of the transmitter is estimated by utilizing the fingerprint database and the measured signal subspace received by the array antenna.

In the case that one transmitter is used, in addition to a desired narrow band signal component, multipath components arrive at the array antenna from distinct directions. For an \( M \)-element array antenna of arbitrary geometry, the received signal can be modeled as

\[
x(d, t) = \left\{ \sum_{p=0}^{P(d)-1} \beta_p(d)a(\theta_p(d)) \right\} s(t) + n(t),
\]

where \( d \) refers to the location of the transmitter (m). \( x(d, t) \) is the \( M \times 1 \) received signal vector at location \( d \), \( s(t) \) is the signal from only one transmitter, \( n(t) \) is the \( M \times 1 \) noise vector with independent identically distributed (i.i.d) Gaussian distribution and is independent from the signal, \( P(d) \) is the number of paths at location \( d \), \( p \) is the path index among multipaths with \( p = 0 \) being the line-of-sight (LOS) path and \( a(\theta_p(d)) = [a_1(\theta_p(d)), \ldots, a_M(\theta_p(d))]^T \) is a steering vector toward \( \theta_p(d) \) direction. Its components are given by

\[
a_m(\theta_p(d)) = \exp\left(j2\pi f_c \tau_m(\theta_p(d))\right)
\]

for an arbitrary reference point. \( \tau_m(\theta_p(d)) \) is the time delay of the \( m \)th element relative to the reference element on \( \theta_p(d) \), \( f_c \) is the center frequency, and the superscript \( T \) denotes the transpose. The steering vector is normalized as

\[
a^H(\theta_p(d))a(\theta_p(d)) = 1,
\]

where the superscript \( H \) denotes the complex conjugate transpose. \( \mathbf{h}(d) \) is defined as

\[
\mathbf{h}(d) = \frac{1}{\gamma(d)} \sum_{p=0}^{P(d)-1} \beta_p(d)a(\theta_p(d)),
\]

where \( \beta_p(d) \) is the complex value representing the multipath coefficient of the \( p \)th path at location \( d \) and \( \gamma(d) \) is the amplitude of the multipath coefficient at location \( d \), so that \( \mathbf{h}(d) \) is normalized.

The output covariance matrix can be written as

\[
\mathbf{R}(d, t) = E[\mathbf{x}(d, t)\mathbf{x}^H(d, t)],
\]

where \( E[\cdot] \) represents the statistical expectation. However in practice, only an estimate of \( \mathbf{R} \) is used and is defined as

\[
\hat{\mathbf{R}}(d, \tilde{t}) = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}(d, t_n)\mathbf{x}^H(d, t_n),
\]

where \( \tilde{t} = \frac{1}{N} \sum_{n=1}^{N} t_n \); \( n \) is the snapshot index.

According to (1), only one large eigenvalue is observed at the eigenvalue distribution obtained from the correlation matrix while other eigenvalues have small values which are close to the noise power. Therefore, the signal subspace has a single dimension. For its eigendecomposition, the correlation matrix can be written as

\[
\mathbf{R}(d, t) = \left[ \sigma_p^2(d, t) + \sigma_n^2 \right] \mathbf{v}_s(d)\mathbf{v}_s^H(d) + \sigma_n^2 \mathbf{v}_{n}(d)\mathbf{v}_{n}^H(d),
\]

where \( \sigma_p^2(d, t) \) is the total received signal power, \( \sigma_n^2 \) is the noise power per element, \( \mathbf{v}_s(d) \) spans the signal subspace and \( \mathbf{v}_{n}(d) \) spans the noise subspace, and \( \{ \mathbf{v}_s(d) \mathbf{v}_{n}(d) \} \) is an unitary matrix. It is noted that \( \mathbf{v}_s(d) \propto \mathbf{h}(d) \).

3. Location Fingerprint Techniques

3.1 An Overview of the Location Fingerprint Technique

The basic concept of the location fingerprint technique is briefly explained in this subsection. By using the fingerprint technique, the location of the mobile terminal can be estimated by comparing the spatial signature with the previously recorded database of known signal-location information through pattern matching algorithms as shown in Fig. 1.

The key points of this technique are the selection of the spatial signature and the method in constructing the database. In this work for the former, the signal subspace obtained by the array antenna is utilized as the spatial signature which works well for the location estimation. For the latter, a new method in constructing the database is proposed. As previously mentioned, the major limitation of this technique is due to the fingerprint database construction. To achieve an accurate location, a very detailed spatial resolution of the database is required. This is time consuming and requires a lot of effort. Alternatively, a new database needs to be reconstructed when the propagation environments change. For feasibility in real applications, fewer data observations in making the database is more preferable. Therefore, the new method in constructing the database is proposed to overcome the above limitation and its performance is compared with previous works which are explained in the following subsection.

For the common framework of the fingerprint techniques considered in this work, the measured signal

![Fingerprint Database](image)

Fig. 1 Location fingerprint localization.
subspace obtained by the array antenna, $v_{s,\text{test}}$, is employed to estimate the transmitter location and will be called the ‘test data’ from here on. Suppose that $\{f^H(d_1), f^H(d_2), \ldots, f^H(d_k)\}$ is a set of $L$ fingerprint vectors which has a one-to-one mapping to a set of the locations $\{d_1, d_2, \ldots, d_k\}$, and the location index is denoted by $k$ ($k = 1, 2, \ldots, L$). Then the estimated location can be obtained by

$$
\hat{d} = \arg \max_{d_k} |f^H(d_k) v_{s,\text{test}}|^2. \tag{7}
$$

The location which provides the best match is represented as the estimated location.

3.2 Straightforward Approaches to Constructing the Fingerprint Database

Several approaches to constructing the fingerprint database can be considered. The authors used two straightforward approaches to compare the performances with the proposed method in this paper. The first constructed database is the geometrically calculated fingerprint database and the second one is the measured signal subspace database.

3.2.1 Geometrically Calculated Fingerprint Database

The fingerprint database can be constructed using the signal information obtained from site-specific geometrical calculations. A well-known tool is the ray tracing which can determine the signal information such as RSS, AOA, TOA, etc., and then these signal information can be used as a fingerprint [8], [11]. As mentioned, one transmitter is used in this work, therefore the direct (LOS) signal is assumed to be received at the array antenna. In the case of the LOS condition, the AOA which is a primitive signal information determined from the geometrical calculation can be employed as a fingerprint of which the database has a continuous spatial resolution. Moreover, the signal subspace coincides with the steering vector toward AOA in a single source scene. Thus the location of transmitter can be estimated by comparing the measured signal subspace (the test data) with the steering vectors toward AOA from known positions stored in the database. For simplicity, the beamforming spectrum can be used to show the similarity between them; it is expressed as

$$
P_{bf}(\theta_0(d_k)) = |a^H(\theta_0(d_k))v_{s,\text{test}}|^2, \quad \tag{8}
$$

where $a(\theta_0(d_k))$ is the steering vector toward $\theta_0(d_k)$ direction. $\theta_0(d_k)$ is the AOA of the LOS signal at the transmitter location $d_k$.

3.2.2 Measured Signal Subspace Fingerprint Database

The researches for the mobile localization using the subspace fingerprint database were presented in [8], [9]. This kind of database uses the subspaces obtained from the discrete measurement points.

When $v_s(d_k)$ is defined as the measured signal subspace at location $d_k$, which is used as a fingerprint, the location of the transmitter can be estimated by matching the test data $v_{s,\text{test}}$ of that transmitter with the fingerprint in the database (referring to Eq. (7)). The location which provides the best match is represented as the estimated location.

4. A Novel Approach in Constructing a Fingerprint Database: Interpolated Measured Signal Subspace Fingerprint Database

For the measured signal subspace fingerprint mentioned in the previous section, the signal subspace for discrete measurement points is used as the fingerprint. Therefore, the location estimation can only be done discretely. In order to estimate the location at any point, an approach in regenerating a continuous spatial signature by using the interpolation of the signal subspace for discrete measurement points is proposed and explained in this section.

4.1 Model of the Signal Subspace

Assuming that the signal subspace measured by the receiving array antenna consists of the combination of the direct component and multipath component when the transmitter is located in the LOS condition. Also assuming that the multipath component is very small. Here, the steering vector calculated by AOA information can be considered as the subspace spanned by the direct component, while the orthogonal complement space of the steering vector can be considered as the space spanned by the multipath component. Generally, it is difficult to calculate the subspace spanned by the multipath component without actual observed signals, hence, the measured signal is necessary to model the signal subspace. Then, the modeled signal subspace at the measured location $d_k$ is denoted by $\bar{v}_s(d_k)$ and can be expressed as

$$
\bar{v}_s(d_k) = |c(d_k)| a(\theta_0(d_k)) + r(d_k), \quad \tag{9}
$$

where $a(\theta_0(d_k))$ is the steering vector toward $\theta_0(d_k)$ direction. $\theta_0(d_k)$ is the AOA of the LOS signal at the transmitter location $d_k$. $c(d_k)$ is a complex coefficient of $a(\theta_0(d_k))$ obtained by

$$
c(d_k) = a^H(\theta_0(d_k))v_s(d_k), \quad \tag{10}
$$

$v_s(d_k)$ is the measured signal subspace at the measured location $d_k$. $r(d_k)$ is the multipath component and can be obtained by the subspace spanning the orthogonal complement space of the steering vector,

$$
r(d_k) = (1 - P_a(d_k))v_{s,p}(d_k), \quad \tag{11}
$$

where $P_{a}(d_k)$ is projection matrix to the steering vector defined as

$$
P_a(d_k) = a(\theta_0(d_k))a^H(\theta_0(d_k)), \quad \tag{12}
$$

and $v_{s,p}(d_k)$ is the phase-adjusted signal subspace defined as
The interpolated signal subspace can then be obtained from
\[ \mathbf{V}_s(d_k) = \frac{|c(d_k)|}{c(d_k)} \mathbf{V}_s(d_k), \]
so that the reference phase of the steering vector is zero.

Note in Eq. (9) that the AOA of the LOS signal is obtained from geometrical calculation because it can guarantee the accurate direction of the LOS signal. Moreover, the multipath component is assumed to be very small (\(|c(d_k)| \gg |\mathbf{r}(d_k)|\)).

4.2 Interpolation of the Signal Subspace for the Construction of a Continuous Fingerprint Database

Given the interpolated location \(d\) be in between two adjacent measured locations \(d_k\) and \(d_{k+1}\), i.e., \(d_k < d < d_{k+1}\) and assuming that there is a high similarity between multipath components at two adjacent locations. The interpolated signal subspace at location \(d\) is denoted by \(\hat{\mathbf{V}}_s(d)\) and can be written as
\[ \hat{\mathbf{V}}_s(d) = |\hat{c}(d)| [a(\hat{\theta}_0(d)) + \hat{\mathbf{r}}(d)], \]
where \(|\hat{c}(d)|\), \(\hat{\theta}_0(d)\), and \(\hat{\mathbf{r}}(d)\) are the real coefficient, the arriving angle and the multipath component at the interpolated location \(d\), respectively. The linear interpolation is employed to obtain each parameter by interpolating from those at the measured locations \(d_k\) and \(d_{k+1}\). For example, the interpolated multipath component at location \(d\) can be obtained by
\[ \hat{\mathbf{r}}(d) = \frac{\mathbf{r}(d_{k+1})(d-d_k) + \mathbf{r}(d_k)(d_{k+1}-d)}{d_{k+1}-d_k}. \]

\(\hat{\theta}_0(d)\) and \(|\hat{c}(d)|\) can be interpolated in the same manner. The interpolated signal subspace can then be obtained from \(|\hat{c}(d)|\), \(\hat{\theta}_0(d)\), and \(\hat{\mathbf{r}}(d)\) (refer to Eq. (14)) and further used to construct a continuous fingerprint database. The transmitter location is then estimated by matching its measured signal subspace with the interpolated signal subspaces in the database (refer to Eq. (7)). Since the database has the continuous fingerprint, the transmitter location can be continuously estimated.

4.3 Limitations on the Spatial Sampling Interval

For the interpolation of two adjacent locations, the appropriate distance of adjacent locations has to be decided. From one location to another, the phase of the LOS and multipath changes or rotates. The appropriate distance or spatial sampling interval is the distance of adjacent locations that does not cause phase ambiguity of the phase rotation due to the movement from one location to another. If the difference of the phase rotation of the LOS components and the phase rotation of the multipath components exceed \(\pi\), phase ambiguity occurs (for ex. \(\pm \pi = \pm 3\pi, \pm 5\pi, \ldots\)) such that the actual phase rotation cannot be known. If this happens, the multipath component cannot be correctly interpolated. To know the maximum spatial sampling interval, consider two transmitters with the phase rotation of LOS components be \(\Psi_{\text{LOS}}\) and the phase rotation of the multipath components be \(\Psi_{\text{Mul}}\). The spatial sampling interval can be examined subject to the constraint
\[ |\Psi_{\text{LOS}} - \Psi_{\text{Mul}}| < \pi. \]
\(\Psi_{\text{LOS}}\) can be determined by
\[ \Psi_{\text{LOS}} = \frac{2\pi(d_{\text{LOS1}} - d_{\text{LOS2}})}{\lambda}, \]
where \(d_{\text{LOS1}}\) and \(d_{\text{LOS2}}\) are the distances of LOS components traveling from both transmitters to the array antenna and can be geometrically calculated. \(\lambda\) is the signal wavelength. In the same manner, \(\Psi_{\text{Mul}}\) can be determined by
\[ \Psi_{\text{Mul}} = \frac{2\pi(d_{\text{Mul1}} - d_{\text{Mul2}})}{\lambda}, \]
where \(d_{\text{Mul1}}\) and \(d_{\text{Mul2}}\) are the distances of multipath components traveling from both transmitters to the array antenna via the multipath source. By assuming that the multipath source is known, these distances can be geometrically calculated.

5. Experiment System and Setup

5.1 Equipment for Experiment

The effectiveness of this technique was verified by an outdoor experiment. The experiment was conducted in Yokosuka Research Park (YRP), Yokosuka, Japan, for a case study. The uniform linear array (ULA) shown in Fig. 2 was used as the receiving antenna. Only the eight middle elements of the ULA were used, while the two remaining elements at both ends were used as dummies to reduce the pattern distortion of the array. The specifications of the receiving array antenna are listed in Table 1. The transmitting antenna was a \(\lambda/4\) monopole antenna and was placed on the roof of a car. The signal was transmitted at a center frequency of 2.335 GHz. The data received from the array antenna were downconverted to an intermediate frequency (IF) of 450 kHz, then digitized with a sampling rate of 2 MHz, and further downconverted to baseband. After calibration using the amplitude and phase compensation technique [12] and decimation in time, these data with 1250 total samples and 62.5 kHz sampling rate were used for AOA and signal subspace estimation. The average received signal-to-noise ratio was 29.25 dB.
Table 1 Specifications of the receiving antenna.

<table>
<thead>
<tr>
<th>Antenna array</th>
<th>Horizontally aligned ULA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements (M)</td>
<td>8 active elements and 2 dummy elements</td>
</tr>
<tr>
<td>Element spacing (d)</td>
<td>0.5 λ</td>
</tr>
<tr>
<td>Antenna type</td>
<td>Vertically polarized microstrip</td>
</tr>
</tbody>
</table>

Fig. 3 The location map of the transmitters and the receiving array antenna.

(SNR) ratio was about 38 dB.

5.2 Measurement Setup

The array antenna was mounted at a height of about 33 m on top of the YRP No. 1 building. The transmitter was placed at known locations under LOS condition along the street and is depicted by black dots in the location map shown in Fig. 3. Each adjacent measured location was 5 m apart. The distance between the transmitter and the array antenna was approximately 340 m when the AOA was 0 degrees. The covered measurement length is 150 m, so 31 measurements were made in total. The field measurements were conducted twice by starting from 0 m to 150 m and then repeated again from 0 m. The received data of the first field measurement were used for the database construction and those of the second field measurement were used for localization to evaluate the performance of the proposed technique.

6. Results and Discussion

6.1 Estimated Location Results Using the Geometrically Calculated (AOA) Fingerprint Database

Figure 4 shows the AOAs from geometrical calculation versus the location. These AOAs were used to construct the database. Since the ULA is used in this work, the steering vector of the ULA is utilized as a fingerprint and can be expressed as

$$a(\theta_0) = \begin{bmatrix} e^{-j\frac{\pi l}{\lambda} \sin \theta_0} \\ e^{-j\frac{2\pi l}{\lambda} \sin \theta_0} \\ \vdots \\ e^{-j\frac{(M-1)\pi l}{\lambda} \sin \theta_0} \end{bmatrix}^T, \quad (19)$$

when the center of the array is used as the reference point and $l$ is the inter-element spacing. Note that $\theta_0 \equiv \theta_0(dx)$ for a compact form of Eq. (19).

Figure 5 illustrates the result of the estimated location when the geometrically calculated AOA fingerprint database is used. For the test data, the observed signal subspaces for every 5 m were used to estimate the transmitter location. Although the AOA fingerprint database has a continuous spatial resolution, the errors in the estimated locations are still big. This may be because the test data do not contain only the direct LOS component but also contain some multipath components. Different characteristics between the fingerprint and test data can give the error in the estimated location. Therefore, only using the AOA as a fingerprint does not give the accurate estimated location.

6.2 Estimated Location Results Using the Measured Signal Subspace Fingerprint Database

Note that two field measurements were conducted in which the received data of the first field measurement were used to construct the database and those of the second field measurement were used to determine the location of the observed transmitter. From here on, the first field measurement will be designated as the ‘database,’ while the second field measurement will be the test data.

Figure 6 shows a comparison of the estimated location of transmitters using two database with different spatial interval. One database composes of the signal subspaces for every 5 m (i.e. 0 m, 5 m, 10 m, . . . , 150 m) and another composes of the signal subspaces for every 10 m (i.e. 0 m, 10 m, 20 m, . . . , 150 m). For the test data, the signal subspaces for every 5 m were used to estimate the location of the observed
It can be seen from Fig. 6 that the location of transmitter can be estimated accurately only if the location of the observed transmitter is at the exact location in the database. Specifically, the location of all observed transmitters can be estimated correctly for the database with the 5 m spatial interval. For the database with the 10 m spatial interval, the location of the observed transmitter that is shifted by 5 m from the database (i.e., 5 m, 15 m, 25 m, ..., 145 m locations) cannot be estimated correctly because the database does not have fingerprints at those locations. Therefore, the finer the spatial resolution of the database, the more accurate the estimated location. Moreover, it is clearly seen that, by using these two databases, the location can only be estimated discretely.

6.3.1 Spatial Sampling Interval

Based on Eq. (16), the spatial sampling interval without phase ambiguity can be geometrically calculated by assuming the multipath source. In this measurement scenario as shown in Fig. 7, the big building near the measurement area (depicted by the dark gray rectangle) is the most possible source of the multipath component contributing higher power than other multipath sources. The distance between the building and the transmitters of every measured location is approximately 5 m. Hence, the distance of multipath component traveling from each transmitter location to the array antenna via the multipath source can be calculated. The maximum spatial sampling interval between two transmitters related to the used center frequency of 2.335 GHz was then determined to be 10 m. Therefore, the authors decided to use an interval of 10 m.

6.3.2 Estimated Location Results

Here, the signal subspace used for the interpolation is modeled, based on Eq. (9) using the geometrically calculated AOA to assure the accurate direction of the LOS signal. The performance of the interpolated database can be evaluated by comparing the results of the estimated location with those using the measured signal subspace database (or measured database for short). Figure 8 shows a comparison of the estimated locations of transmitters for the measured and interpolated database. The measured database was constructed using the signal subspace with a 10 m interval, i.e., 0 m, 10 m, 20 m, ..., 150 m locations. The interpolated database consists of the signal subspaces interpolated from the measured database. For the test data, the signal subspaces for every 5 m were used to estimate the location. It can be seen from Fig. 8 that the estimated locations using the interpolated database are closer to the true locations than using the measured database. Therefore, this is an advantage of the proposed method in that a small set of measured data can be used to have a fine spatial resolution database, which is more practical in the real situations.

In order to discuss later on the effectiveness of the proposed interpolation technique based on the estimated location result, the location estimation error using the interpolated database is compared with those using the measured database in Fig. 9. The effectiveness of the proposed interpolation technique can be investigated by measuring the similarity between the measured and interpolated signal...
subspaces at the same location using the inner product between them. Note that the authors have the measured signal subspace at every 5 m, and the interpolated database used only the measured signal subspace at 0 m, 10 m, 20 m, ..., 150 m locations. Then, the inner product between the interpolated signal subspace at 5 m, 15 m, 25 m, ..., 145 m locations and the measured signal subspace at 5 m, 15 m, 25 m, ..., 145 m locations can be compared. Figure 10 shows this inner product with 1 denoting the highest similarity. It can be seen that the similarity between the measured and interpolated signal subspaces is very high. This means the interpolated signal subspace can be used instead of the measured signal subspace as a fingerprint and the location can be estimated with small errors as shown in Fig. 9. However, there are relatively low similarities at the 25 m and 145 m locations. Low similarities of these locations result in big estimated location errors. It can be found from Fig. 9 that the estimation errors were less than 5 m for most locations that the authors were satisfied. Thus, under the controlled and assumed conditions of the measurement, the low bound of the similarity of 0.9 is given to achieve the location estimation error of less than 5 m.

As previously mentioned, the multipath component is assumed to be very small compared to the LOS component. If this condition is not satisfied, the interpolated database is not correctly obtained which results in the big error of the estimated location. To identify whether the assumption is satisfied, the Rician Factor $K$ can be used. The Rician Factor $K$ is defined as the ratio of the power of the LOS component to the power of the multipath component. The value of the Rician Factor is a measure of the severity of the multipath effect, with $K$ approaching to zero being the most severe multipath effect. Referring to Eq. (9), the Rician Factor $K$ at location $d$ can be approximated and expressed in decibels as

$$K(d) = 10 \log_{10} \left( \frac{||c(d)||^2}{||r(d)||^2} \right) \text{[dB]}.$$ (20)

The Rician Factor of every measured location is shown in Fig. 11. Except for the 25 m and 145 m locations, it is shown that the $K$ at each transmitter location is high. This means the LOS component is relatively strong compared to the multipath component. Therefore the assumption is satisfied which leads to a high similarity of the interpolated and measured signal subspaces (see Fig. 10), and further results in the small errors in the estimated locations (see Fig. 9). On the other hand, the 25 m and 145 m locations were strongly influenced by the multipath so that the assumption for these locations is not satisfied. Hence, similarities of the interpolated and the measured signal subspaces are relatively low (see Fig. 10), leading to the relatively big errors in the estimated locations (see Fig. 9). Therefore, the proposed method is valid only if this assumption is satisfied. In order to achieve the location estimation error of less than 5 m, the low bound of the Rician Factor of 5 dB is given.

The standard deviation (STD) of estimated location errors using the geometrically calculated AOA, measured and interpolated database for two sets of measured data obtained along the street and at the racing car course [10] are listed in Table 2. Looking at the STD, it is observed that estimating the transmitter location using the interpolated database
is better compared to that of using the AOA and measured database for both environments. This is because the interpolated database has a very fine spatial resolution (compared to the measured database) and characteristics between the fingerprint and the observed signals are more similar (compared to the AOA database). Note that the measured and interpolated database for the street data used a spatial sampling interval ($\Delta$) of 10 m, whereas the racing car course data used $\Delta$ of 100 m. Comparing the results of the racing car course data to those of the street data, it can be found that the finer the measured spatial sampling interval used in constructing the database, the smaller the estimated location error obtained. Specifically, the estimated location for the street data with $\Delta$ of 10 m has a smaller STD than the racing car course data with $\Delta$ of 100 m.

7. Conclusion

This paper proposes a method of the signal subspace interpolation in constructing a location fingerprint database. A continuous database can be obtained by interpolating the measured signal subspaces. This makes the proposed method more practical in real applications. Specifically, the database can easily be reconstructed using a small set of measured data than those of other methods.

From the results of estimated locations, the interpolated database improved the accuracy of the estimated location compared to the geometrically calculated AOA database which also has a continuous spatial resolution. It is implied that although the experiment was done under the LOS condition, not only the direct path was received, but also some other multipaths were received. That is the reason why the big errors of the estimated locations were observed when the AOA database was employed. On the other hand, using the proposed method by assuming that the signal subspace is composed not only of the LOS component, but also the multipath component, the small errors of the estimated locations were obtained. Compared with the measured database, the interpolated database yields location estimation more accurate. This is because of the finer spatial resolution database. Therefore, given the controlled and assumed conditions of the measurement, the proposed technique can be used and the estimation error of less than 5 m can be achieved for most locations.

References

Hiroyuki Tsuji received the B.S., M.S., and Ph.D. degrees from Keio University in 1987, 1989, and 1992, respectively. In 1992, he joined the Communications Research Laboratory (CRL), the forerunner of the National Institute of Information and Communications Technology (NICT), Ministry of Posts and Telecommunications, Japan. In 1999, he was a visiting researcher at University of Minnesota. He has been working in the NICT, Incorporated Administrative Agency, Japan. His research interests are in array signal processing, particularly as applied to communications. He received the IEICE 1996 Young Engineer Award.