LETTER

A New Technique of Reduction of MEI Coefficient Computation Time for Scattering Problems

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SUMMARY In this letter, we propose a new technique that reduces the computation time to derive the MEI coefficients in solving scattering problems by the Measured Equation of Invariance (MEI) methods. Methods that use the MEI technique spend most of the computation time in the integration process to derive the MEI coefficients. Moreover, in the conventional solution process, some repeated computation of metron fields to derive the MEI coefficients is included. To avoid the repeated operations and thus save computation time, we propose a matrix localization technique in computing the MEI coefficients. The time comparison for the computation of MEI coefficients of a cylinder and a sphere is given to verify the CPU time reduction of our new technique.

key words: MEI technique, matrix localization technique, SIE-MEI method

1. Introduction

Methods [1]–[4] that use the Measured Equation of Invariance (MEI) technique [5] to solve the wave scattering problem, generate a sparse linear system and result in minimum memory and CPU time requirements for computing the final matrix. The total computation time of these methods consists of the time required in the integration process to fill the matrix with the MEI coefficients and that for the matrix inversion. Since the matrix is sparse, the time for solving the matrix system is much less than that for generating the matrix elements. Therefore, the computation of MEI coefficients becomes a bottleneck for solving the scattering problem. Many researchers have tried to minimize the computation time in different ways [6]–[14], but it is still necessary to introduce some suitable technique for the MEI coefficient calculation.

In this letter, we propose a Matrix Localization (ML) technique for the reduction of computation time of the MEI coefficients. Although we describe the ML technique only with the Scalar-field approach of the IEMEI (SIE-MEI) method, it can be easily implemented in other methods which use the MEI technique without any significant modification.

2. Scalar-Field Integral Equation of SIE-MEI Method

Let us consider a closed surface ∂V+ of region V+ placed very close to the scatterer and assume that the region includes only a single source which is represented by the equivalent monopole source ρ2 and the dipole moment μ2 as shown in Fig. 1. This leads to the Scalar-field Integral Equation [4]

\[ \int_{\partial V} \left( \varphi_1(r) \rho_2(r) - \frac{\partial \varphi_1(r)}{\partial n} \hat{n} \cdot \mu_2(r) \right) dS = 0, \]

where \( \varphi_1 \) and \( \frac{\partial \varphi_1}{\partial n} \) are the scattered field and its normal derivative, respectively, and \( (\cdot) \) terms represent the equivalent sources near the scatterer. The detailed definition of Eq. (1) can be found in [4].

3. Derivation of MEI Coefficients

3.1 Conventional Technique

In the conventional solution process, SIE-MEI postulates [4] are then applied to Eq. (1). Therefore, the discretized version of a local linear equation for node \( n = 1, 2, \ldots, N \) is [4]

\[ \sum_m \left[ a_{nm} \varphi_1(r_{nm}) - b_{nm} \varphi'_1(r_{nm}) \right] = 0, \]

where \( m = 1, 2, \ldots, M \) are the integration points within the local region, \( a_{nm} \) and \( b_{nm} \) are the MEI coefficients for the \( n \)-th node and its neighboring \( M \) nodes, and \( \varphi_1(r_{nm}) \) and \( \varphi'_1(r_{nm}) \) are the scattered fields and their normal derivatives, respectively, generated by the suitable \( q = 1, 2, \ldots, Q \)-set of secondary sources.

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*Fig. 1* Surface \( \partial V^+ \) of region \( V^+ \) very near to the scatterer.
sources, called metrons [5].

For the \(q\)-th certain metron \(\rho_q(r_n)\), the metron fields are [4]

\[
\varphi_{1,q}(r_{nm}) = \sum_{n=1}^{N} \rho_q(r_n) G(r_{nm}, r_n) \Delta S_n, \quad (3)
\]

\[
\varphi'_{1,q}(r_{nm}) = \sum_{n=1}^{N} \rho_q(r_n) \frac{\partial G(r_{nm}, r_n)}{\partial n} \Delta S_n, \quad (4)
\]

where \(G(r_{nm}, r_n)\) is the 3D free-space Green’s function, \(r_{nm}\) and \(r_n\) are the position vectors of observation and source points, respectively, and \(\Delta S_n\) is the area of the \(n\)-th segment.

Using the abbreviations \(\phi_{q,n,m} \equiv \varphi_{1,q}(r_{nm})\) and \(\phi'_{q,n,m} \equiv \varphi'_{1,q}(r_{nm})\), Eq. (2) is expressed as a local matrix equation [4]

\[
\begin{bmatrix}
\phi_{1,n,1} & \cdots & \phi_{1,n,M} \\
\phi_{2,n,1} & \cdots & \phi_{2,n,M} \\
\vdots & \vdots & \vdots \\
\phi_{Q,n,1} & \cdots & \phi_{Q,n,M}
\end{bmatrix}
\begin{bmatrix}
a_{n1} \\
\vdots \\
a_{nM} \\
b_{n1} \\
\vdots \\
b_{nM}
\end{bmatrix} = 0, \quad (5)
\]

or more concisely,

\[
[CD][ab] = 0, \quad (6)
\]

where \([CD]\) is the \([Q \times 2M]\) matrix of metron fields and their normal derivatives and \([ab]\) is the unknown column vector of MEI coefficients.

In Eq. (5), each metron field \(\phi_{q,n,m}\) or \(\phi'_{q,n,m}\) is obtained by the linear combination of reaction term \(G(r_{nm}, r_n)\) or \(\frac{\partial G(r_{nm}, r_n)}{\partial n}\), multiplied with the metron \(\rho_q(r_n)\), according to Eq. (3) or (4), respectively, which requires \(N\) operations. Accordingly, the local matrix \([CD]\) requires a total of \(Q \times 2M \times N\) operations, to obtain the MEI coefficients for a particular \(n\) node.

This procedure is repeated for each node \((n = 1, 2, \cdots, N)\) of the scatterer surface as shown in Fig. 2, to obtain the sparse matrices \(A\) and \(B\) which are explained in Ref. [4] and consist of \(a\) and \(b\) in Eq. (6).

The sparse matrices are the cyclic band matrices as of Eqs. (16) and (17) in Ref. [15] or scattered sparse matrices with \(M\) nonzero elements in each row depending on the index of the scatterer surface.

Therefore, the conventional solution procedure requires a total of \(Q \times 2M \times N \times N\) operations, to obtain the MEI coefficients for all the nodes.

From the above discussion it is seen that, in the conventional solution technique, localization is introduced before the metron field generations. Hence, the solution procedure requires some repeated operations in the integration process which causes extra computation time. To avoid this type of repeated operation, we propose the matrix localization technique, where localization is introduced after the metron field generation.

### 3.2 Matrix Localization Technique

For the implementation issue, the following “measurement” scheme is considered.

At first, we derive the metron field \(\phi_{q,n,1}\) and its normal derivative \(\phi'_{q,n,1}\), for all \(q = 1, 2, \cdots, Q\) and \(n = 1, 2, \cdots, N\) nodes following the same procedure as of Eq. (5). This produces a \([Q \times 2N]\) dimensional global matrix as

\[
\begin{bmatrix}
\phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,N} & \phi'_{1,1} & \phi'_{1,2} & \cdots & \phi'_{1,N} \\
\phi'_{1,1} & \phi'_{1,2} & \cdots & \phi'_{1,N} & \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,N} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\phi_{Q,1} & \phi_{Q,2} & \cdots & \phi_{Q,N} & \phi'_{Q,1} & \phi'_{Q,2} & \cdots & \phi'_{Q,N}
\end{bmatrix}, \quad (7)
\]

where \(\phi_{q,n,1}\) and \(\phi'_{q,n,1}\) are defined as \(\phi_{q,n}\) and \(\phi'_{q,n}\) respectively, and each of the metron fields requires \(N\) operations. Correspondingly, the full matrix requires a total of \(Q \times 2N \times N\) operations.

Finally, we introduce a selection matrix for the localization as

\[
[S] = \begin{bmatrix}
L & 0 & \cdots & 0 \\
0 & L & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & L & 0 \\
0 & \cdots & 0 & 0 & L
\end{bmatrix}, \quad (8)
\]

where \([S]\) is the \([2N \times 2M]\) matrix of 0’s except some 1’s in the localization part \(L\). The dimension of \(L\) block matrix is \([N \times M]\), which keeps only a single 1 in each column, and the rest are zero. Thus, the total number of 1’s in matrix \(L\) is equal to \(M\), i.e., the number of nodes within the local region. The row position of 1’s in the \(L\) part depends on the position of the localized nodes on the surface, which changes with the change of nodes considered for localization.

As an example, let us consider a sphere whose surface \(S\) is discretized and localized as shown in Fig. 3. Due to the axial symmetry along \(\varphi\), measuring function is taken only in \(\theta\) direction, and the local region \(S_o\) consists of \(M = 3\) segments in the polar direction.

Let us assume that the node \(n\) is considered for the localization and the nodes \(n-1, n,\) and \(n+1\) are
the nodes within the local region, then the localization part \( L \) of the selection matrix becomes

\[
L = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & \cdots & 1 & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & 1 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 1 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\end{bmatrix}
\tag{9}
\]

where \( n - 1 \), \( n \), and \( n + 1 \) represents the row position of 1’s.

Now, by multiplying the global matrix (Eq.(7)) with the selection matrix (Eq.(8)), we can obtain the local matrix equation for the node \( n \) as

\[
\begin{bmatrix}
\phi_{1,n-1} & \phi_{1,n} & \phi_{1,n+1} & \phi_{1,n+1} \\
\phi_{2,n-1} & \phi_{2,n} & \phi_{2,n+1} & \phi_{2,n+1} \\
\vdots & \vdots & \vdots & \vdots \\
\phi_{Q,n-1} & \phi_{Q,n} & \phi_{Q,n+1} & \phi_{Q,n+1} \\
\end{bmatrix}
\begin{bmatrix}
a_{nn-1} \\
a_{nn} \\
a_{nn+1} \\
b_{nn-1} \\
b_{nn} \\
b_{nn+1} \\
\end{bmatrix} = 0,
\tag{10}
\]

which is equivalent to Eq.(5), where \( a_{nn-1}, a_{nn}, a_{nn+1} \) correspond to \( a_{n1}, a_{n2}, a_{n3} \) respectively, and \( b_{nn-1}, b_{nn}, b_{nn+1} \) correspond to \( b_{n1}, b_{n2}, b_{n3} \) respectively, for \( M = 3 \).

In this way, by columnwise cascading, for all \( n \) nodes of the scatterer surface we can solve the local matrix simultaneously, to obtain the sparse matrix \( A \) and \( B \) of MEI coefficients.

Therefore, the total computation time required in our proposed technique is only \( Q \times 2N \times N \), the time required for the global matrix generation. This means that the matrix localization technique can reduce the computation time to \( \frac{1}{M} \) of the conventional one.

4. Time Comparison

In the SIE-MEI method, the dominant part of the computation is the integration process to obtain the MEI coefficients. By using the conventional solution technique, the integration process requires \( O(2QM^2N^2) \) operations, alternatively, the proposed matrix localization technique requires only \( O(2QN^2) \) operations, where \( Q \) is the number of metrons, \( M \) is the number of integration points in the local region, and \( N \) is the total number of integration points on the scatterer surface. Therefore, we can reduce the computation time to \( \frac{1}{M} \) of the conventional one by using the proposed localization technique.

In the two-dimensional case, \( M \) is assumed to be 3 but in the three-dimensional case, \( M \) should be more than 3 depending on the surface complexity. Hence, by using the matrix localization technique more than 3 times the computation time can be saved, which gives measurable time savings on the overall CPU time requirements.

Figures 4(a) and (b) show the time comparison for the 2D cylinder and the 3D sphere, respectively, for different radii. In the comparison, the piecewise linear discretization for the 2D cylinder and the rectangular patch discretization for the 3D sphere and the pulse-basis point matching method are used with ten points in one wavelength. The number of integration points in the local region are considered to be 3. In both of these situations, we try to maintain the same parameters for
a fair comparison.

From the comparison it is seen that, the time required in the conventional solution technique is larger than the time required in the proposed matrix localization technique. As $M$ or the size of the scatterer increases, we can save more computation time in our matrix localization technique.

5. Conclusion

In the conventional technique, the localization process is implemented at the beginning of the computation which requires some repeated operation in the integration process. But in the matrix localization technique, it is applied after the integration process. This modification reduces the computation time without affecting the scattering results.

The ML technique can also be applied to other methods which use the MEI technique for the wave scattering computation. To validate our comments, we are working on implementing this technique in the FDMEI and the Integral Equation formulation of MEI (IE-MEI) method.

References