Efficient Multi-channel Wideband Spectrum Sensing Technique Using Filter Bank

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Abstract—As a growing interest in the cognitive radio technology has been taken based on the idea of opportunistic spectrum use, the spectrum sensing technique for dynamic channel environment is more and more important to ensure that cognitive radios would not interfere with primary systems while it achieves reasonable throughput opportunistically. Architecture based on filter bank allows to sense multi-channels simultaneously with low spectrum leakage from adjacent channels thanks to efficient implementation of bandpass filters. Therefore it is now attracting much interest. In this paper, we propose a framework of multi-channel sensing architecture based on polyphase DFT filter bank followed by energy detector with minimum complexity. The sensing performance is evaluated by numerical simulation as well as theoretical analysis comparing with the conventional periodogram spectral estimator and time domain sequential sensing.

Index Terms—cognitive radios, spectrum sensing, filter bank, energy detector

I. INTRODUCTION

As the spectrum scarcity is recognized as the most critical problem of wireless communication due to the current and ever-increasing demand of wireless systems, cognitive radio (CR) technology that allows the operation of unlicensed secondary devices within licensed primary band is now attracting much interest as a candidate solution [1]. Some frameworks to identify spectrum white space have been proposed so far in order that it ensures that cognitive radios would not interfere with primary user (PU) systems while it achieves reasonable throughput. In current situation, it is recognized as a most promising scheme where the spectrum sensing in secondary systems does not require any modification of current primary systems nor additional infrastructure to provide information of white space [2]. However, spectrum sensing is quite challenging task because secondary system should be capable of detecting the primary signal of very low SNR (signal to noise power ratio) due to multipath or shadowing fading with high reliability. In addition, the spectrum sensing should monitor multiple channels simultaneously and identify white spaces for opportunistic use across the wide range of target frequency band.

There are previous studies to detect primary users’ signals on single channel have been mainly considered. In this approach, a tunable narrowband bandpass filter at the RF front-end were used to sense single narrow frequency band at a time. In reality, the techniques that sense multiple channels simultaneously and identify multiple available bands at a time are necessary for opportunistic frequency use but have rarely been discussed. The multiple channel sensing should be introduced for realization of CR systems as mentioned in [3] where the wideband spectrum sensing strategy for optimum throughput was proposed. For wideband operation, the RF front-end requires wideband architecture and the spectrum sensing usually involves the estimation of the power spectral density (PSD) of the wideband signal.

Most basic way for the PSD estimation based on short time discrete Fourier transformation (DFT) is periodogram spectrum estimator (PSE). The tradeoff between spectrum resolution and dynamic range caused by the sidelobe of the PSE window’s frequency response is a traditional problem in PSE method. In the multichannel sensing, the spectrum leakage from adjacent channels due to the sidelobe usually brings erroneous decision. The other advanced techniques include multi-taper (MT) method, filter bank (FB), and etc [4], [5]. A significant advantage of filter bank among them is that it enables efficient implementation of bandpass filters by polyphase decomposition of prototype filter. MT method and filter bank based sensing have been compared in [5] precisely, and FB turned out more promising from lower computational cost.

Now, there are some existing filter bank applications for CR spectrum sensing as [6–8], respectively. However, in these literatures, the performance of filter bank has not been discussed enough yet. In fact the discussion about practical sensing performance from the implementation point of view is necessary. Therefore, this paper makes detail investigations into the detection and false alarm probabilities of the filter bank by theoretical analysis and numerical simulation. In addition, the feasibility in complexity compared with conventional PSE and time domain sequential sensing is discussed.
II. SYSTEM MODEL

In time domain, each narrow band channel can be modeled based on the following hypothesis as

\[
\begin{align*}
\mathcal{H}_0 : r_m(n) &= v_m(n) & \text{: signal absence} \\
\mathcal{H}_1 : r_m(n) &= h_m(n) \cdot s(n) + v_m(n) & \text{: signal presence}
\end{align*}
\]

where \( r_m(n) \) denotes received passband signal of \( m \)-th primary user (PU) channel. \( s(n) \) and \( v_m(n) \) denote a transmitted signal and the band limited additive white Gaussian noise (AWGN) process at \( m \)-th PU channel. The linear time varying characteristics \( h_m(n) \) of the propagation channel is herein assumed to be constant without loss of generality. The received signal including all channels’ components \( x(n) = \sum_{m=1}^{M} r_m(n) \) is the input to the system and the spectrum of each PU channel is detected by spectrum sensor as illustrated in Fig.1.

III. SPECTRUM SENSING USING FILTER BANK

A. DFT Filter Bank

Filter bank is usually categorized into two types of synthesis and analysis filter banks. We introduce the latter case to extract signal component of each subband from the wideband RF signal. The concept of analysis filter bank is an array of bandpass filters as shown in Fig.2. Each bandpass filter is the frequency-shifted version of a lowpass filter \( H_0(\omega) \) with impulse response of \( h(n) \) with the length \( L \) which is called a prototype filter. The mathematical expression of \( m \)-th band of the filter bank is given by

\[
h_m(n) = h(n) \cdot W_m^{mn} \quad (m = 0, 1, ..., M-1),
\]

where \( W_M^w = e^{-j \frac{\pi}{2w}} \) and \( M \) is the number of subbands. Let \( x(n) \) denote the input signal sampled by the analog to digital converter (ADC) at sampling rate of \( f_s = 1/T_s \) with the length \( N \) as

\[
x(n) = x(t)|_{t=nT_s} \quad (n = 0, 1, ..., N-1).
\]

Then, the output of \( m \)-th subband can be written as

\[
Y_m(n) = \sum_{i=0}^{L-1} h_m(i) \cdot x(n-i) = \sum_{i=0}^{L-1} h(i) \cdot W_M^{-mn} \cdot x(n-i).
\]

If the relationship of \( L = PM \) is assumed, the \( M \)-polyphase decomposition of the input signal and prototype filter can be applied. By using polyphase decomposition and down-sampling by \( M \)-to-1, Eq.(4) can be rewritten by

\[
Y_m(n) = \sum_{l=0}^{M-1} \sum_{p=0}^{P-1} h(pM+l) \cdot x(n-pM-l) \cdot W_M^{ml}.
\]

Now, we can introduce the polyphase decomposed versions of \( h(n) \) and \( x(n) \) as

\[
h_l(k) = h(n)|_{n=km+l}, \quad x_l(k) = x(n)|_{n=km+l}.
\]

By substituting Eqs.(6) and (7) into Eq.(5), we can obtain the output spectra as

\[
Y_m(k) = \sum_{l=0}^{M-1} \sum_{p=0}^{P-1} h_l(p) \cdot x_l(k-p) \cdot W_M^{-ml} = \sum_{l=0}^{M-1} y_l(k) \cdot W_M^{-ml},
\]

where \( y_l(n) \) denotes the \( l \)-th phase filtered output. The mentioned relationships are well described in Fig.3, which called polyphase DFT filter bank. As it can be seen, the discrete inverse Fourier transform (DFT) can be applied in order to compute the output signal at each subband in Eq.(8). When \( M \) is power of two, fast Fourier transform (FFT) can be utilized to increase the computation performance.

Although \( M \)-branch filter bank consists of \( M \) subbands, in case with even number \( M \) only \((M-1)\) subbands within Nyquist zone \((w < \pi)\) can be utilized for sensing. In this paper the minimum complexity design where the user channels correspond exactly to the subbands, namely \( M = 2(N_{PU} + 1) \), is taken into consideration utilizing bandpass filter. A requirement of prototype filter is low energy leakage in order to obtain higher sensing accuracy. In this paper, we employ a prolate sequence filter which is optimal in minimizing sidelobe energy [12].

B. Sensing by Energy Detection

The simplest method to determine whether the channel is active or not is to compare the accumulated received signal power (energy) with predetermined threshold, which is called energy detection based on \( \chi^2 \) hypothesis test. We extend this idea to multi-channel sensing by filter bank. In order to apply energy detection to each channel, the individual filter bank output is followed by an energy detector. The spectrum of
each PU channel is decomposed into $Y_m(n)$ by filter bank as explained in previous section. The test statistics are given by

$$T_m = \sum_{k=l_m}^{u_m} |Y_m(k)|^2,$$

where $l_m$ and $u_m$ denote lower and upper bound indices of the output subband for $m$-th user channel. And $N$ denotes the number of samples to be used for sensing per channel. The detection rule with threshold $\gamma_m$ is as

$$\begin{cases} T_m \geq \gamma_m & \text{signal present} \\ T_m < \gamma_m & \text{signal absent} \end{cases}.$$

The proposed sensing architecture of the filter bank can be expressed as shown in Fig.4. The composite signal of multiple PU channels is down-converted into intermediate frequency. Then, the passband signal is sampled by wideband ADC and fed into the proposed processing block.

IV. PROPERTY OF MULTI-CHANNEL ENERGY DETECTOR

A. DFT Filter Bank

The theoretical sensing performance of proposed architecture shown in Fig.4 will be derived in this section. We assume that $x(n)$ is Gaussian random process with variance $\sigma_n^2$ and zero mean under the $H_0$ hypothesis. Then, the filter bank output $Y_m(k)$ is also Gaussian random process with zero mean and variance $\text{Var}[Y_m] = G\sigma_n^2$ where $G$ denotes the processing gain of the filter as $G = \sum_{l=0}^{L-1} h^2(l)$, since the filter bank is a linear operation. The test statistics in Eq.(9) can be written by

$$T_m = \eta G \frac{\chi^2_N}{2}$$

where random variable $\eta$ follows the $\chi^2_N$ distribution since the real and imaginary parts of $Y_m$ are i.i.d. random processes. According to the central limit theorem, the test statistics $T_m$ can be approximated by the normal distribution asymptotically as the sample number becomes larger as

$$T_m \approx N \left( S_m NG\sigma_n^2, S_m NG^2\sigma_n^4 \right),$$

where $S_m = u_m - l_m + 1$ denotes the number of subbands for $m$-th user channel, and the number of sample per channel $N = \lfloor \frac{N}{D} \rfloor$ where $\lfloor \cdot \rfloor$ denotes an operator that rounds a real number towards minus infinity. Subsequently, the probability of false alarm can be calculated by

$$P_{FA,m} = Q \left( \frac{\gamma_m - S_m NG\sigma_n^2}{\sqrt{S_m NG^2\sigma_n^4}} \right),$$

where the detection threshold $\gamma_m$ for each channel is usually obtained by predetermined $P_{FA,m}$.

$$P_{D,m} = Q \left( \frac{Q^{-1}(P_{FA,m}) \sqrt{S_m NG\sigma_n^2 - S_m \bar{N}\sigma_n^2}}{\sqrt{S_m \bar{N}(\sigma_n^2 + \sigma_g^2)}} \right).$$

It can be found that the prototype filter effect which is an advantage of the filter bank processing cannot appear in theoretical performance $P_{D,m}$. Hereafter, without loss of generality, we assume that $P_{FA} = P_{FA,m}$ and all channel conditions are identical so that $\gamma = \gamma_m$, and hence the overall theoretical sensing performances are supposed to be same that $P_D = P_{D,m}$.

B. Periodogram Spectral Estimator (PSE)

On the other hand, the conventional technique of the spectrum estimation is periodogram by block DFT. It is basically identical to the proposed architecture except for the window function $g(k)$ ($k = 0, 1, ..., M-1$) and the downsampling parameter $D$ which is determined depending on the overlapping ratio, e.g., the cases that $D = M$ and $D < M$ denote no overlapping and with overlapping, respectively. This fact allows analyzing theoretical sensing performance in the same way as the proposed architecture. Thus,

$$P_D = Q \left( \frac{Q^{-1}(P_{FA}) \sqrt{S_m \bar{N}\sigma_n^2 - S_m \bar{N}\sigma_n^2}}{\sqrt{S_m \bar{N}(\sigma_n^2 + \sigma_g^2)}} \right),$$

where the number of sample per channel $\bar{N} = \lfloor \frac{N-M}{D} \rfloor + 1$.

C. Time Domain Sequential Sensing

A very traditional way of sequential sensing where each channel selected at a time by analog filters in RF front-end is sensed sequentially by tuning the local oscillator frequency like a radiometer. It results in system overhead to change the operation parameters (e.g. oscillator’s frequency) from channel to channel. The necessity of tuning time is undesirable when the requirement of the sensing period is very strict due to fast variation of the channel status [9]. Fundamentally the statistical properties are same both in frequency domain and time domain sensing. If $N$ signal samples are totally used to compute $M/2$ subbands within Nyquist zone in frequency domain, $2\lfloor \frac{N}{M} \rfloor$ samples should be used for each subband.
in time domain sensing. Therefore, considering $S_m$ as the bandwidth of the signal of interest, the theoretical performance of the probability of detection of each channel is identically

$$P_D = Q \left( \frac{Q^{-1}(P_{FA}) \sqrt{S_m \bar{N}_m \sigma_s^2 - S_m \bar{N}_m \sigma_s^2}}{\sqrt{S_m \bar{N}(\sigma_s^2 + \bar{N}\sigma_s^2)}} \right),$$  \hspace{1cm} (16)

where $\bar{N} = [\frac{\bar{N}}{\bar{N}}]$. Eq.(16) is also identical to Eqs.(14), and (15) with $D = M$.

V. NUMERICAL SIMULATION

In order to examine the sensing performance in more practical situation, numerical simulation using ISDB-T signal, Japanese digital broadcasting system was conducted. The parameters are presented in Table I. We assumed that the composite RF signal with multiple PU channels is down-converted to IF centered at $f_s/4$. The sampled signal by wideband ADC is the input of the processing. The activity of primary users is represented by channel state vector, $V$ as

$$V = [v_1, v_2, \cdots, v_{N_{PU}}],$$  \hspace{1cm} (17)

where the value $v_k = 1$ when the $k$-th channel is active and $v_k = 0$ otherwise. This simulation assumes three PU channels with equal bandwidth which are equally spaced in frequency domain.

First, we observed the performance of $P_D$ for the 1-st user’s channel only in case of $V = [1, 0, 0]$. Again, we evaluated $P_{FA}$ in three different cases made by $V = [v_1 = 0, v_2, v_3]$ except for $[0, 0, 0]$, respectively. In this simulations, the SNR was defined as $10 \log_{10} \frac{P_s}{P_n}$ where $P_s$ and $P_n$ mean the signal and noise power within the single channel, respectively. The noise component was evenly added throughout the whole band so that total noise power becomes $(1 + N_{PU})P_n$. Then, the sampled signal for simulation, namely $x(n)$, was obtained. The $x(n)$ was processed by using the mentioned sensing methods in section IV. Then, the detection rule was applied in each method, respectively. In filter bank and PSE, a prolate sequence filter with the minimum configuration of $M = 8$ subbands and the length $L = 64$ and rectangular window with $M = 8$ subbands were used, respectively. The frequency responses of the filters corresponding to the PU 1 are illustrated on the input signal spectrum in Fig.5.

The simulation result for the probability of detection $P_D$ is shown in Fig.6. Ideally $P_D$ performance should be identical in all methods. However, the PSE shows slightly worse performance than filter bank due to extra noise reception by the broad DFT response as shown in Fig.5. In addition, slight improvement can be recognized comparing with PSE due to the spectral leakage of the DFT, although there is no difference between filter bank and PSE from the theoretical expression in Eqs.(14) and (15).

On the other hand, Fig.7 shows the probability of false alarm $P_{FA}$ with respect to the power ratio of the neighboring PU channel signals to the noise. It is found that the PSE has large $P_{FA}$ and this trend gets more significant at high neighboring channels’ SNR due to the power leakage from them. It is observed that $P_{FA}$ gets significantly degraded in cases that the neighboring channels are present ($V = [0, 1, 0]$ and $V = [0, 1, 1]$). In fact, the $P_{FA}$ of PSE improves as the spectrum resolution increases but the complexity of the processing also increases. As a matter of fact, the filter bank shows almost same $P_{FA}$ at any SNR in any cases. We can see that $P_{FA}$ characteristic of the filter bank is a significant advantage in multi-channel spectrum sensing.

VI. COMPLEXITY

The motivation to introduce filter bank is twofold. First, it enables the implementation of $M$ subband filters at the cost of only single prototype filter, which can be utilized to extract multiple channels efficiently. Second, polyphase decomposition can reduce computation rate (operations per unit time) by factor $M$. We introduce the number of multiplications as a criterion of complexity. Table II shows the comparison between filter bank and PSE. Each term of the complexity represents implementation cost of the prototype filter (or window) and DFT in order. In reality, bandpass filter generated by DFT has broad bandwidth as found in Fig.5, and hence higher resolution should be required for reliable performance. The PSE with $S > 8$ and guard band should be

\begin{tabular}{|c|c|}
\hline
**TABLE I** & **SIMULATION PARAMETERS.**
\hline
\hline
Signal & ISDB-T mode 3
\hline
No. of primary users, $N_{PU}$ & 3 channels
\hline
Carrier modulation & 64 QAM
\hline
Number of subcarriers & 5,617
\hline
Carrier spacing & 0.992 MHz
\hline
Channel bandwidth & 5.57 MHz
\hline
Guard band & 0.43 MHz
\hline
Guard interval & 126 $\mu$s
\hline
Symbol period & 1,008 $\mu$s
\hline
Sampling frequency & 48 MHz
\hline
Prototype filter & prolate sequence filter ($L = 64$)
\hline
FFTs points, $M$ & 8
\hline
Signal length, $N$ & 10,000 samples
\hline
False alarm rates ($P_{FA}$) & 0.05
\hline
SNR & $-25 \sim 3$ [dB]
\hline
Trials & 1,000
\hline
\end{tabular}
necessary to obtain reasonable performance comparing with the filter bank.

The filter bank spectrum sensor has naturally more complexity than PSE due to the cost of the filter implementation. Figure 8 shows the comparison where the ratio of the number of multiplications in the filter bank to that of DFT (herein, $\log_2(M)$) in PSE is plotted with respect to the log of the number of coarse subbands $\log_2(M)$. For example, with simulation parameter $M = 8$, $L = 64$ and $S = 1$ the circuit complexity of the proposed filter bank architecture is 2.75 times higher than PSE. However the filter bank outperforms PSE as $S$ and $M$ get larger. Although larger $L$ provides better performance of prototype filter, $L$ should be chosen by careful consideration of the trade-off between implementation cost and performance.

VII. CONCLUSION

In this paper, the performance of the filter bank followed by the energy detector was evaluated in comparison with conventional PSE and time domain sequential sensing. We derived theoretical probabilities of detection and false alarm, respectively. In addition, the simulation is conducted with multiple channels of ISDB-T OFDM signals to verify the performance in more practical situation. The simulation result reveals the significant advantage of the filter bank spectrum sensing comparing with conventional PSE; both probabilities of false alarm and detection could be improved. Also, the circuit complexity of the architecture is quantitatively examined. The implementation of the filter bank outperforms conventional methods. However, the prototype filter length of the filter bank should be carefully chosen based on the trade-off between cost and performance.

REFERENCES


![Fig. 6. Probability of detection ($P_D$).](image)

![Fig. 7. Probability of false alarm ($P_{FA}$).](image)

![Fig. 8. Comparison of complexity between filter bank ($L = 64$) and PSE ($S = 1, 2, 4, 6, 8$).](image)