Experimental Evaluation of a SAGE Algorithm for Ultra Wideband Channel Sounding in an Anechoic Chamber

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Abstract—Results of experiments in an anechoic chamber that assessed the reliability of UWB channel sounding system based on a deterministic approach with SAGE algorithm is presented. The system could resolve and detect 10 [deg] separated waves in angle domain, which was near the resolution limit. In the delay domain, 0.67 [ns] separated waves could be resolved where the relation between the bandwidth of subband and spectrum estimation was discovered. The behavior of the algorithm for the detection of incident waves which are within the resolution limit was examined. The spherical incident wave model was applied for estimating the parameters of incident waves. This model could drastically reduce the number of spurious paths compared to the plane wave model.

I. INTRODUCTION

Ultra Wideband (UWB) communication is believed to be a promising scheme for future broadband communication and attracts a lot of interests recently. For commercial use, UWB communication can be applied to the wireless personal area network (WPAN) in indoor environments. In terms of the propagation environment, however, the indoor environments are much complexed compared to the outdoor channel. Therefore, the knowledge of spatio-temporal channel characteristics and waveform distortion effects are essential for the efficient design of UWB communication systems, such as Rake receivers and pulse generation timing in transmitters.

We have been developing a UWB channel sounding system in a deterministic way [1]. The system measures the spatial transfer function distribution using a vector network analyzer (VNA) in conjunction with a X-Y scanner. Then measured data are applied to the SAGE algorithm which derives the direction-of-arrival (DOA), time-of-arrival (TOA) and spectrum of each path. In the ray-based approach of propagation channel, the behavior of the algorithm must be evaluated when incident waves are within the inherent resolution of the algorithm. Moreover, the estimation of the number of incident waves is a significant problem for the accurate result of the algorithm. These aspects are specific for the ray-based channel estimation and are not considered in the moment based analysis, e.g., angle or delay spread.

In this paper, results of the experiment conducted in an anechoic chamber are presented. The experiment assessed the spatio and temporal resolution of the proposed UWB channel sounding system.

II. SAGE ALGORITHM FOR UWB CHANNEL ESTIMATION

The SAGE algorithm estimates parameters and spectrum of each path from measured data in VNA. Details of the algorithm can be found in [1]. In this paper, a mathematical expression of the spherical incident wave model, that was not covered in [1], is presented.

A. Spherical Incident Wave Model

Spherical incident wave model can be expressed mathematically by using azimuth arrival angle $\phi_l$, elevation arrival angle $\psi_l$, delay time $\tau_l$ and curvature radius from source point, $R_l$. In the definition of the coordinate depicted in Fig. 1, the position vector from center of horizontal array to source point, $r_{0,l}$ is expressed as

$$r_{0,l} = \begin{bmatrix} R_l \cos \psi_l \sin \phi_l \\ R_l \cos \psi_l \cos \phi_l \\ R_l \sin \psi_l \end{bmatrix}.$$ 

As well, the position vector of the antenna, $r_a$, which is distant from center of the array by $d_{x,k_1}$ and $d_{y,k_2}$ in $x$, $y$ coordinates respectively, is

$$r_{a,k_1,k_2} = \begin{bmatrix} d_{x,k_1} \\ d_{y,k_2} \\ 0 \end{bmatrix}.$$
Therefore, transfer function of frequency index \( k_3 \) observed at the antenna position index \((k_1, k_2)\) can be expressed in the following way,

\[
y_{k_1,k_2,k_3} = \sum_{i=1}^{L} \{s_{i,k_3} \times \exp \left\{ -j \frac{2\pi f_{k_3}}{c} \left( |r_{0,i} - r_{a,k_1,k_2}| - R_i \right) \right\} \times \exp \left\{ -j2\pi f_{k_3} \tau_i \right\} \}
\]

where \( L \) impinging waves exist, \( s_{i,k_3} \) denotes a reference transfer function observed at the center of array, \( n_{k_1,k_2,k_3} \) is additive white Gaussian noise with zero mean and power of \( \sigma^2 \). \( d_{x,k_1}, d_{y,k_2} \) can be denoted by antenna spacing \( d \) and the number of sampling in \( x, y \) coordinate \( K_1, K_2 \),

\[
d_{x,k_1} = \left( \frac{K_1}{2} - k_1 \right) d \quad (k_1 = 1, 2, \ldots, K_1) \\
d_{y,k_2} = \left( \frac{K_2}{2} - k_2 \right) d \quad (k_2 = 1, 2, \ldots, K_2).
\]

If \( K_3 \) points of frequency sweeping is conducted within a certain bandwidth in which antennas and propagation characteristics are assumed to be constant, we can obtain \( K_1 K_2 K_3 \) transfer functions. In order to simplify the notation, obtained transfer functions are vectorized as

\[
y = \begin{bmatrix} y_{1,1,1} & y_{1,1,2} & \cdots & y_{1,1,K_3} & \cdots & y_{1,2,1} & \cdots & y_{1,K_2,K_3} & \cdots & y_{1,1,1} & y_{1,1,2} & \cdots & y_{1,K_2,K_3} \end{bmatrix}^T.
\]

By using this notation, spherical array mode vector for the \( l \)th wave can also be expressed as

\[
a_l = \begin{bmatrix} a_{1,1,1,l} & a_{1,1,2,l} & \cdots & a_{1,1,K_3,l} & a_{1,2,1,l} & \cdots & a_{1,K_2,K_3,l} & a_{2,1,1,l} & \cdots & a_{2,1,K_3,l} & \cdots & a_{K_1,1,K_3,l} & a_{K_1,K_2,k_3,l} \end{bmatrix}^T,
\]

\[
a_{k_1,k_2,k_3,l} = \exp \left\{ -j \frac{2\pi f_{k_3}}{c} \left( |r_{0,l} - r_{a,k_1,k_2}| - R_l \right) \right\} \times \exp \left\{ -j2\pi f_{k_3} \tau_l \right\}.
\]

Finally, the obtained transfer function vector \( y \) yields the following equation,

\[ y = As + n, \tag{1} \]

where \( s \) is a reference transfer function vector which contains parameters of incident waves,

\[
s = \begin{bmatrix} \alpha_1(f_{k_3})r_{1}(f_{k_3})D(f_{k_3}, \phi_1, \psi_1) & \cdots & \alpha_3(f_{k_3})r_{3}(f_{k_3})D(f_{k_3}, \phi_3, \psi_3) \end{bmatrix}^T \in \mathbb{C}^L, \tag{2}
\]

and \( A \) is a spherical array mode matrix,

\[
A = \begin{bmatrix} a_1 \ a_2 \cdots \ a_L \end{bmatrix} \in \mathbb{C}^{K \times L}. \tag{3}
\]

\( K = K_1 K_2 K_3 \) and \( n \in \mathbb{C}^K \) is a noise vector.

By using the notation described above, log-likelihood function for spherical incident wave model can be derived exactly into the same equation as the plane wave model [1]. Note that the dimension of the search in which log-likelihood function is maximized increases by introducing \( R \) in the spherical incident wave model.

### B. Considerations on SAGE Algorithm

Here we point out some considerations of the SAGE algorithm for reliable UWB channel estimation and channel modeling.

1) Estimation of the Number of Incident Waves: In the maximization of the log-likelihood function, successive interference cancellation (SIC) type procedure, which detects the strongest wave and removes it in a sequential manner, is employed [3]. The number of waves can be determined by the detected waves whose power is greater than the noise level. In our processing, SIC iteration stops until the number of waves reaches a predefined number. This implementation enables us to obtain stable result of parameters and spectrum without being affected by the estimation of the number of waves.

2) Subband Processing: In order to estimate a frequency dependent spectrum for each path, the whole bandwidth is divided into several subbands. Log-likelihood of whole bandwidth is defined as a sum of log-likelihood in the subbands. This process reduces the distortion effect of amplitude and phase caused especially
by antennas when the parameters of incident waves are estimated. At the same time, however, resolution of delay time decreases due to the subband processing, then it is necessary to choose the appropriate bandwidth of the subband. The bandwidth of subband is chosen according to the spatio-temporal resolution of the considered wave and of the wave that is the nearest around the considered wave, as shown in Table I. For two waves that can be resolved in the angle domain, a subband, in which antenna and propagation characteristics is assumed to be constant, is enough. In contrast, when two incident waves within the inherent resolution of SAGE, the bandwidth of the subband should be more than a fraction of $\Delta \tau$, where $\Delta \tau$ is the delay time difference of two waves.

After the parameter estimation of each wave, the spectrum is extracted by using smoothing procedure of log-likelihood. The phase and amplitude components of frequency $f_c$ are derived by taking an average of log-likelihood over bandwidth $B$ whose center frequency is $f_c$. Bandwidth of the subband chosen according to the criteria shown in Table I is sufficient for $\Delta \psi$. In this case, it may happen that the spectrum is extracted is impossible in the lower and higher frequency region of the bandwidth where enough bandwidth of subband cannot be assured. This phenomenon occurs in the region of $\frac{1}{2\Delta \tau}$ near the lowest and highest frequency.

3) Deconvolution of Antenna Effects: It is necessary to deconvolve the antenna characteristics from the measurement result for an antenna independent channel model, because the SAGE algorithm described in [1] does not have a process for antenna deconvolution. We can deconvolve the antenna characteristics in the search of SAGE algorithm by introducing the antenna transfer function into the spherical array mode vector. However, we can also evaluate the antenna characteristics by using the spectrum of each wave derived from SAGE algorithm. Using the latter makes it possible to implement the antenna deconvolution easier, but note that the group delay characteristics of the measurement antenna must be smooth, otherwise estimation error of the delay time will be significant. This is because we assumed in the SAGE algorithm that each propagation ray has frequency independent delay time.

### III. Experiment in an Anechoic Chamber

We assessed the proposed system in terms of spatio-temporal resolution in an anechoic chamber. Note that antenna characteristics are already deconvolved for all the spectrum depicted in the result.

#### A. Evaluation of Spatio-Temporal Resolution

Spatio-temporal resolution measurement system is depicted in Fig. 3. By using the 3-dB power divider in Tx side, two point sources whose relative position are arbitrary configured are realized. Antennas are UWB monopole antennas whose fluctuation of group delay are less than 0.1 [ns] in the considered bandwidth [4].

1) Evaluation of Spatial Resolution: In the experiment, element spacing of the synthesized array realized by a X-Y positioner are set to 48 [mm], which are equivalent to 0.49$\lambda$ in the lowest frequency 3.1 [GHz] but larger than the half wavelength in the higher frequency region. In this case, we can avoid the aliasing in angle domain by calculating the log-likelihood with the whole frequency domain data but the resolution of angle domain is restricted by the lower frequency. In general, fixed physical aperture length corresponds to smaller electrical aperture in the lower frequency region which leads to wider beamwidth and lower resolution in angle domain. In our experiment,
TABLE III
RESULT OF PARAMETERS ESTIMATION: TWO WAVES WHOSE TOAS ARE SAME BUT 10 DEGREE DIFFERENT DOAS.
THEORETICAL VALUES ARE SHOWN IN ( ).

<table>
<thead>
<tr>
<th>Azimuth (deg)</th>
<th>Elevation (deg)</th>
<th>Delay (ns)</th>
<th>Curvature radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.30</td>
<td>2.20</td>
<td>16.65</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(3.43)</td>
<td>(16.70)</td>
</tr>
<tr>
<td>#2</td>
<td>10.10</td>
<td>0.80</td>
<td>16.65</td>
</tr>
</tbody>
</table>

Fig. 4. Result of spectrum estimation: two waves whose TOAs are same but 10 [deg] different DOAs.

the electrical aperture length is $4.5\lambda$ and approximately corresponds to 10 [deg] resolution in angle domain. We confirmed this property by using the system depicted in Fig. 3. Two waves whose delay time and arrival elevation angles are the same but with azimuth angles of 10 [deg] difference were realized and examined to be separated by SAGE algorithm. The bandwidth of subband used was 800 [MHz]. The estimated parameters of the two waves are shown in Table III where theoretical values obtained from Tx and Rx positions are also included. Fig. 4 is the extracted spectrum of each path. The magnitude and phase refers to the absolute attenuation of free space and deviation from free space phase rotation, respectively. Theoretical characteristics, derived from the extended Friis’ transmission formula consists of real and imaginary parts. The estimated spectrum in Fig. 4 agrees well with the theoretical value, but we can observe linearly deviated phase components from the theoretical line. These linear phase components correspond to certain delay time, and in this case, about 0.01 [ns]. This is equivalent to 3 [mm] in free space and can be attributed to the error of the antenna positions in the experiment. These results indicate that the algorithm can resolve two waves that are close to the resolution limit in angle domain.

2) Evaluation of Temporal Resolution: Since the system uses 7.5 [GHz] bandwidth, inherent resolution in the delay domain is calculated to be 0.13 [ns]. However, in the experiment of assessing the temporal resolution, we examined the detection of 0.67 [ns] separated waves in

TABLE IV
RESULT OF PARAMETERS ESTIMATION: TWO WAVES WHOSE DOAS ARE SAME BUT 0.67 NANosecond different TOAs.
THEORETICAL VALUES ARE SHOWN IN ( ).

<table>
<thead>
<tr>
<th>Azimuth (deg)</th>
<th>Elevation (deg)</th>
<th>Delay (ns)</th>
<th>Curvature radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.30</td>
<td>1.90</td>
<td>16.00</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(3.58)</td>
<td>(16.03)</td>
</tr>
<tr>
<td>#2</td>
<td>0.30</td>
<td>0.80</td>
<td>16.65</td>
</tr>
</tbody>
</table>

Fig. 5. Result of parameters estimation: two waves whose DOAs are same but different TOAs of 0.67 [ns].
order to confirm the relation between the bandwidth of subband and spectrum estimation limit observed at the lower and higher frequency, as described in section II. The bandwidth of subband was chosen as the fraction of delay time difference, 1.5 [GHz]. The results of parameters and spectrum estimation were shown in Table IV and Fig. 5, respectively. We could confirm that the system separated spectrum in the higher and lower frequency region of \( \pm \frac{1}{2\pi} \). Note that we could exactly observe this phenomenon even when we change the difference of delay time, \( \Delta \tau \).

B. Considerations on Resolution and Accuracy of the Estimation

1) Spherical Incident Wave Model: In short range wireless communications, especially in indoor environments, it is often the case that the signal model error becomes significant if plane wave incident model is considered because physical phenomenon of propagation seems to be a spherical wave incidence. The signal model error appears in the result of SAGE algorithm as spurious paths, which cannot be identified in real environments. Spurious path is a significant problem in the detection of weaker paths as well as for the accurate modeling of the channel. Here we show one example, which estimates the waves incident from azimuth 0 [deg] (#1) and 15 [deg] (#2) using the plane and spherical incident wave models. Note that the radiation power of point source at 15 [deg] is 20 [dB] weaker compared to the point source at 0 [deg] with the use of an attenuator. Fig. 6 is a log-likelihood spectrum in which the search for finding #2 is conducted after the successful detection and removal of #1. For the plane incident wave model, residual components after the extraction of #1 appear in \( \pm 5 \) [deg] and their response is larger than that of the true source at 15 [deg]. In this case, the SAGE regards the peak at 5 [deg] as the second strongest incident wave and a spurious path is produced. The SAGE detects a spurious path for the third wave as well and the true source will only be estimated in the fourth iteration. On the contrary, the spectrum using the spherical incident wave model in Fig. 6 indicates that the response of the true source is enhanced compared to the residual component of #1 and #2 can correctly be detected as the second strongest wave. We confirmed this kind of spurious reduction by using the spherical incident wave model for the real environment estimation. Note that the spherical incident wave model can also deal with plane wave incidence because plane wave corresponds to the spherical wave whose curvature radius approaches infinity. In other words, curvature radius gives us a lot of insight about the source distribution. However, in the estimation of large curvature radius, we must bear in mind that the gradient of log-likelihood is so small that the accuracy of the curvature radius estimation may seriously degrade.

2) Spatio-temporal Resolution: The behavior of the algorithm for the detection of incident waves which are within the resolution limit is essential for the reliability of real environment result. In the case where both angle and delay difference are smaller than the inherent resolution, ex. 5 [deg] angle difference and same delay time in our experiment, two incident waves could be resolved but biased estimation of spectrum was observed.

IV. CONCLUSION

We assessed the reliability of proposed UWB channel sounding system in an anechoic chamber. The system could resolve and detect 10 [deg] separated waves in angle domain, which was near the resolution limit. In the delay domain, 0.67 [ns] separated waves could be resolved where the relation between the bandwidth of subband and spectrum estimation was discovered. The behavior of the algorithm for the detection of incident waves which are within the resolution limit was examined. The use of spherical incident wave model enabled us to reduce the spurious paths compared to the plane wave model, as well as to estimate the distribution of sources.

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