# Comprehensive Calibration for MIMO System

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#### Abstract

In this paper a comprehensive calibration procedure for MIMO system is proposed. This technique is able to eliminate effects of the following three unwanted components from MIMO measurement data, i.e. multiple transmitters characteristic, multiple receivers characteristic, and mutual coupling in array antenna elements. The numerical simulation proves that there is an improvement in the DOA estimator employing the proposed calibration technique compared with that employing conventional calibration or without any calibration.

## Keywords

MIMO, calibration, multiple transmitters characteristic, multiple receivers characteristic, mutual coupling in array antenna elements

# INTRODUCTION

MIMO system, employing multiple antennas both at transmitter and receiver, has been getting much interest among communication and propagation engineers. In MIMO communication systems, we can increase the channel capacity without expanding the required frequency band [1][2], if the channel state information (CSI) can be measured accurately. In MIMO propagation measurement systems, we can estimate channel parameters such as directions-of-arrival (DOA) and directions-of-departure (DOD) [3][4] from the accurately measured CSI. In both applications, the CSI should be measured accurately, and it is preferable not to include the effects of multiple transmitters and receivers characteristics, and mutual coupling in array antenna elements.

In this paper, a novel comprehensive calibration procedure, that can eliminate the effects of three unwanted components from the measured data, is proposed. The three unwanted components are multiple transmitters characteristic (MTC), multiple receivers characteristic (MRC), and effect of mutual coupling in array antenna elements (MCAA). Since MTC and MRC are time variant due to temperature and aging, the calibration procedure should be done on site. To accomplish this aim, we employed transmitting signal as the reference signal to measure the MTC and MRC as in [5]. In addition to this, we tried to measure the MCAA also by using this reference signal. In the real system, however, it is just a feedback signal, which includes all of the effects of MTC, MRC, and MCAA, can be measured. Therefore it is very difficult to distinguish the effect of MCAA from the feedback signal as described in [6]. To combat this problem, an active antenna structure was newly introduced. This structure actively controls the impedance of antenna elements that enables port extension technique as in a network analyzer. With this structure the MTC, MRC, and MCAA can be extracted from the feedback signal. Finally the MTC and MCAA form the compensation matrix for transmission, and the MCAA and MRC form that for reception.

## TARGETS FOR CALIBRATION

First of all, it is better to clarify what are the targets for calibration. Given a *m*-element array antenna with scattering matrix of  $S \in C^{m \times m}$ , *m*-branch transmitter with transfer matrix  $H_s \in C^{m \times m}$ , and *m*-branch receiver with transfer matrix  $H_r \in C^{m \times m}$  as in Fig.1, the compensation matrix for transmission  $C_s \in C^{m \times m}$  and that for reception  $C_r \in C^{m \times m}$  are expressed as

$$C_s = H_s^{-1} (I + S)^{-1}$$
 (1)

$$C_r = (I - S)^{-1} H_r^{-1},$$
 (2)

where (I + S) and (I - S) describe the effect of MCAA for transmission and reception.

Derivation of these equations are given as follows. Given an impedance matrix Z of array antenna and an impedance matrix  $Z_L$  of loads, we would like to describe the relationship between RF transmitting voltage vector  $v_s^{RF}(t) \in C^m$  or RF receiving voltage vector  $v_r^{RF}(t) \in C^m$  and resulting excited voltage vector  $v^{ant}(t) \in C^m$  on array antenna. If we define an excited current vector on array antenna as  $i^{ant}(t) \in C^m$ , the following two circuit equations are established.

$$\boldsymbol{v}^{ant}(t) = \boldsymbol{Z}\boldsymbol{i}^{ant}(t) + \boldsymbol{v}_r^{RF}(t)$$
(3)

$$\boldsymbol{v}^{ant}(t) = -\boldsymbol{Z}_L \boldsymbol{i}^{ant}(t) + \boldsymbol{v}_s^{RF}(t) \tag{4}$$

Eliminating  $i^{ant}(t)$  from Eq.(3) and (4), the desired relationship can be obtained as

$$v^{ant}(t) = \frac{1}{2}(I - S)v_r^{RF}(t) + \frac{1}{2}(I + S)v_s^{RF}(t),$$
 (5)

where we used the transformation

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{Z}_L)^{-1} (\boldsymbol{Z} - \boldsymbol{Z}_L).$$
(6)

From Eq.(5), we could understand that (I + S) and (I - S) describe the effect of MCAA for transmission and reception. From another point of view, it is noted that the effect of MCAA can be measured either with the known  $v_s^{RF}(t)$  or  $v_r^{RF}(t)$ . The calibration technique with  $v_r^{RF}(t)$  is established for example in [6]. But it is not suitable for the

on-site calibration, since it requires the known  $\boldsymbol{v}_r^{RF}(t)$  as in an anechoic chamber.

In addition to the effect of MCAA, the MTC and MRC are considered as follows. If the transmitters and receivers are well isolated, that means  $H_s$  and  $H_r$  are diagonal matrices, the RF signals  $v_s^{RF}(t)$  and  $v^{ant}(t)$  and baseband signals  $v_s(t) \in C^m$  and  $v_r(t) \in C^m$  are related as

$$\boldsymbol{v}_{\boldsymbol{s}}^{RF}(t) = \boldsymbol{H}_{\boldsymbol{s}}\boldsymbol{v}_{\boldsymbol{s}}(t) \tag{7}$$

$$\boldsymbol{v}_r(t) = \boldsymbol{H}_r \boldsymbol{v}^{ant}(t). \tag{8}$$

Substituting these into Eq.(5) and with some duplex, the next two equations are established, that describe the target system for transmission and reception.

$$\boldsymbol{v}^{ant}(t) = \frac{1}{2}(\boldsymbol{I} + \boldsymbol{S})\boldsymbol{H}_s \boldsymbol{v}_s(t)$$
(9)

$$\boldsymbol{v}_r(t) = \frac{1}{2} \boldsymbol{H}_r(\boldsymbol{I} - \boldsymbol{S}) \boldsymbol{v}_r^{RF}(t)$$
(10)

Finally we could derive the compensation matrices described in Eq.(1) and (2).

In these equations, it should be noted that  $H_s$  and  $H_r$  are slowly varying due to temperature and aging. Therefore onsite calibration is indispensable. It is our goal to estimate  $C_s$  and  $C_r$  in such a situation.



Figure 1. Target MIMO system.

# **COMPREHENSIVE CALIBRATION**

To accomplish on-site calibration, the transmitting signal vector  $\boldsymbol{v}_s(t)$  is used as reference signal, and then the received feedback signal vector  $\boldsymbol{v}_r(t)$  is expressed as

$$\boldsymbol{v}_r(t) = \boldsymbol{H}_r \boldsymbol{S} \boldsymbol{H}_s \boldsymbol{v}_s(t) = \boldsymbol{H}_{off} \boldsymbol{v}_s(t).$$
(11)

In this formulation, it is impossible to distinguish S from the feedback signal unless some special techniques are introduced.

In our proposal, we have introduced the active antenna structure as illustrated in Fig. 2. It controls the impedance of antenna element actively. In this case, the antenna impedance can be seen  $Z(\text{on}) = \infty$  with switch on and Z(off) = Zwith switch off from the circuit side. With switch on, the



Figure 2. An example of active antenna structure.

following additional information can be measured

$$\boldsymbol{v}_r = \boldsymbol{H}_r \boldsymbol{H}_s \boldsymbol{v}_s = \boldsymbol{H}_{on} \boldsymbol{V}_s. \tag{12}$$

With respect to  $H_s$  and  $H_r$ , relative expressions  $\hat{H}_s$  and  $\hat{H}_r$  are enough for calibration purpose. By using Eq. (11) and Eq. (12),  $\hat{H}_s$ ,  $\hat{H}_r$ , and S are solved as follows. Firstly, for convenience, the following matrices are defined

$$\boldsymbol{H}_{s} = \operatorname{diag}(\boldsymbol{H}_{s})\operatorname{diag}(\boldsymbol{H}_{s}^{-1})^{T}$$
(13)

$$\tilde{\boldsymbol{H}}_r = \operatorname{diag}(\boldsymbol{H}_r)\operatorname{diag}(\boldsymbol{H}_r^{-1})^T, \qquad (14)$$

where the operator diag selects the diagonal components from the matrix and puts them into a vector. These matrices include the relative values of all of the combination of elements in the transfer matrices. Therefore we can reconstruct the relative transfer matrices  $\hat{H}_s$  and  $\hat{H}_r$  from  $\tilde{H}_s$  and  $\tilde{H}_r$ easily. In practice, since the estimates of  $\tilde{H}_s$  and  $\tilde{H}_r$  include noise components, it is better to choose the well conditioned components, which have high SNR, to reconstruct  $\hat{H}_s$  and  $\hat{H}_r$ . By using these expressions,  $\hat{H}_s$  and  $\hat{H}_r$  are calculated as

$$\tilde{H}_s = .\sqrt{M_2./M_1} \to \hat{H}_s$$
 (15)

$$\tilde{\boldsymbol{H}}_r = .\sqrt{\boldsymbol{M}_{2.} \times \boldsymbol{M}_1} \to \hat{\boldsymbol{H}}_r$$
 (16)

where  $.\sqrt{}$ ,  $.\times$ , and ./ mean element-wise calculation, and memories  $M_1$  and  $M_2$  are

$$\boldsymbol{M}_1 = (\boldsymbol{H}_{off})./(\boldsymbol{H}_{off})^T \tag{17}$$

$$\boldsymbol{M}_{2} = \operatorname{diag}(\boldsymbol{H}_{on})(\operatorname{diag}(\boldsymbol{H}_{on}^{-1}))^{T}.$$
 (18)

By using these estimates the scattering matrix  $\boldsymbol{S}$  is absolutely estimated as

$$\boldsymbol{S} = \hat{\boldsymbol{H}}_{r}^{-1} (\boldsymbol{H}_{off}) \hat{\boldsymbol{H}}_{s}^{-1} \hat{\boldsymbol{H}}_{on} \boldsymbol{H}_{on}^{-1}, \qquad (19)$$

where  $\hat{H}_{on} = \hat{H}_r \hat{H}_s$ .

Finally the compensation matrices,  $C_s$  and  $C_r$ , could be formed by using  $\hat{H}_s$ ,  $\hat{H}_r$  and S.

## **IMPLEMENTATION ISSUES**

We should clarify what are needed to implement the proposed calibration method. In the TDD system, m circulators and m PIN diode switches are required as in Fig.2. On the other hand, in the FDD system m additional transmitters for calibration of receivers, m additional receivers for calibration of transmitters, and m more PIN diode switches are needed as illustrated in Fig.3 in addition to the m circulators and m PIN diode switches.



#### Figure 3. Additional hardware for comprehensive calibration in FDD system.

These considerations are just examples. Other configurations are also possible.

# NUMERICAL SIMULATION

In the numerical simulation, the proposed calibration algorithm was used in a DOA estimator. The DOA estimation error was evaluated with and without calibration. The ES-PRIT [8] algorithm was employed as a DOA estimator, and a simple single wave and no noise situation was assumed as illustrated in Fig.4. Table 1 shows the MIMO system configuration. The scattering matrix S, that indicates the amount of MCAA, is calculated by using the method of moments (MoM) as in [7]. The power of S matrix is visualized in Fig.5. If there is no MCAA, only the diagonal elements have meaningful power. From this figure, we could understand the effect of MCAA, especially the characteristic difference between the end elements and the central elements of the array antenna. In these conditions the DOA estimation errors were evaluated as in Fig. 6 by changing the DOA setting angle calculated from the broadside of the ULA. From this result, we could confirm the effectiveness of comprehensive calibration that could achieve almost negligible estimation error. It should also be noted that the MRC calibration as in [5] does not reduce the DOA estimation error substantially. In another simulation, we could confirm almost the same result in a DOD estimator. This result indicates the validity of the proposed calibration in transmission as well as in reception.

	Table 1.	Simulation	condition
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array antenna	8-element ULA	
antenna element	half-wave dipole	
element spacing	half-wave	
active switch	ON: 1[ $\Omega$ ] OFF: $\infty$ [ $\Omega$ ]	
amp. variation in MTC	$\pm 3[dB] UR$	
phase variation in MTC	$\pm 5$ [deg] UR	
amp. variation in MRC	±3[dB] UR	
phase variation in MRC	$\pm 5$ [deg] UR	

ULA: Uniform Linear Array UR: Uniform Random



Figure 4. Simulation scenario for DOA estimation.

#### CONCLUSION

The comprehensive calibration procedure for MIMO system was proposed. This technique can eliminate the effect of the following three components from the MIMO measured data, multiple transmitters characteristic, multiple receivers characteristic, and effect of mutual coupling in array antenna elements. The numerical simulation proved that there is an improvement in the DOA estimator employing the proposed calibration technique compared with that employing conventional calibration or without any calibration.

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Figure 5. Image visualization of scattering matrix of target array antenna.

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Figure 6. DOA estimation error with and without calibration.