

Modified Array Calibration for Precise Angle-of-Arrival Estimation

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Agenda

- Introduction
- Signal model
- Multiple Signal Classification (MUSIC) algorithm
- Experiment
- Array calibration methods
- Results
- Conclusion

Introduction - 1

Objective:

- To propose array calibration methods using the real measurement data
 - Amplitude and phase compensation technique
 - Phase approximation based on least square problem
- To evaluate the effectiveness of the proposed calibration methods by estimating AOAs.

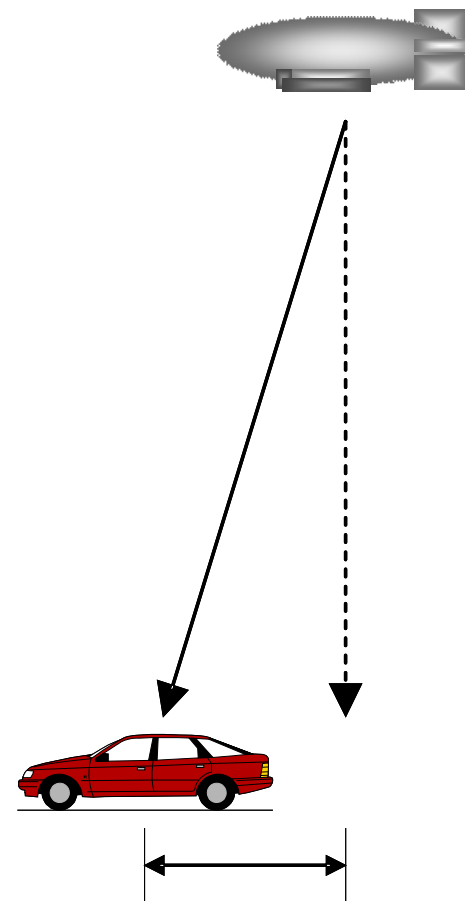
Introduction - 2

Motivation:

- To estimate AOAs precisely
 - 10m location accuracy the same as GPS

Antenna Height	Required Resolution
4km	0.14 degrees
20km	0.03 degrees

- High resolution of AOA estimation is required.
- To obtain a high performance of AOA estimation, the antenna calibration is one of the problems.



Signal Model

- Consider K -narrowband signal sources impinging at an M -element ULA, an array output vector can be expressed as

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{s}(t)$ is the signal vector

$\mathbf{n}(t)$ is the noise vector

$\mathbf{A}$ is the steering matrix defining by

$$\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]_{M \times K}$$

$\mathbf{a}(\theta)$ is the steering vector defined by

$$\mathbf{a}(\theta) = [1 \dots \exp\{-j2\pi d(M-1)\sin(\theta)/\lambda\}]^T$$

$\mathbf{C}$ is the the $M \times M$ calibration matrix.

MUSIC Algorithm

The estimate output covariance matrix:

$$\hat{\mathbf{R}} = E[\hat{\mathbf{x}}(t) \hat{\mathbf{x}}^H(t)] = \tilde{\mathbf{A}} \mathbf{P} \tilde{\mathbf{A}}^H + \sigma^2 \mathbf{I}$$

where $\mathbf{P} = E[\mathbf{s}(t) \mathbf{s}^H(t)]$ is the source covariance matrix
 $\sigma^2 \mathbf{I} = E[\mathbf{n}(t) \mathbf{n}^H(t)]$ is the noise covariance matrix
and $\tilde{\mathbf{A}} = \mathbf{C} \mathbf{A}$.

The eigendecomposition of $\hat{\mathbf{R}}$ can be expressed as

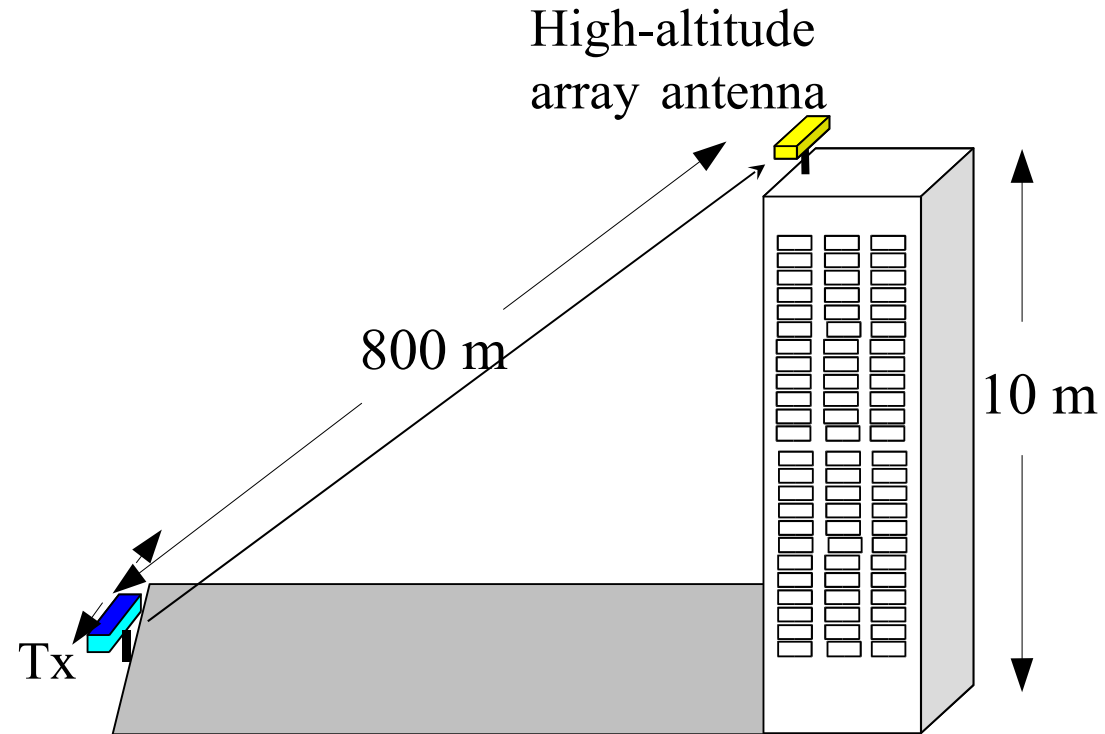
$$\hat{\mathbf{R}} = \hat{\mathbf{U}}_s \hat{\mathbf{\Lambda}}_s \hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n \hat{\mathbf{\Lambda}}_n \hat{\mathbf{U}}_n^H$$

where $\hat{\mathbf{U}}_s = [\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_K]$ is the matrix containing signal eigenvectors
and $\hat{\mathbf{U}}_n = [\hat{\mathbf{u}}_{K+1}, \dots, \hat{\mathbf{u}}_M]$ is the matrix containing noise eigenvectors

Since $\hat{\mathbf{U}}_n^H \tilde{\mathbf{a}}(\theta) = 0$, the AOA can be estimated by
locating the peaks of the MUSIC spatial spectrum given by

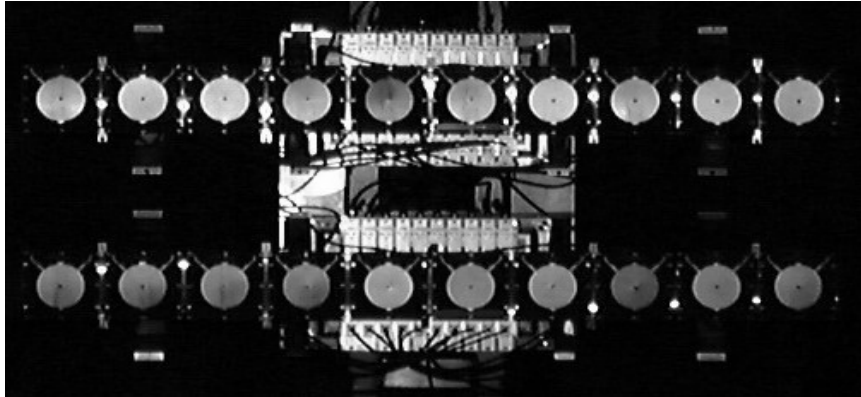
$$P_{MUSIC}(\theta) = \frac{(\tilde{\mathbf{a}}^H(\theta) \tilde{\mathbf{a}}(\theta))}{(\tilde{\mathbf{a}}^H(\theta) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \tilde{\mathbf{a}}(\theta))}$$

Experiment -1



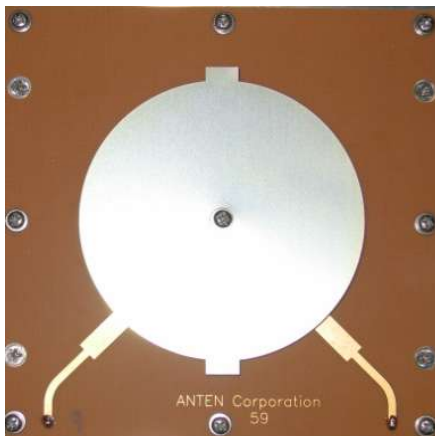
- The transmitted signal:
 - GMSK modulated signal with $f_c = 1.74$ GHz and power = 30 dBm.
- Moving position of Tx in which AOA at Rx are -6, -5, -2, -1, -0.5, -0.2, -0.1, -0.05, 0, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, and 6 deg.

Experiment - 2



- Specifications of the receiving antenna
 - Number of elements : 10
 - Spacing of elements : 0.8λ
 - Antenna element : patch element
 - Antenna gain : 7 dBi (front)

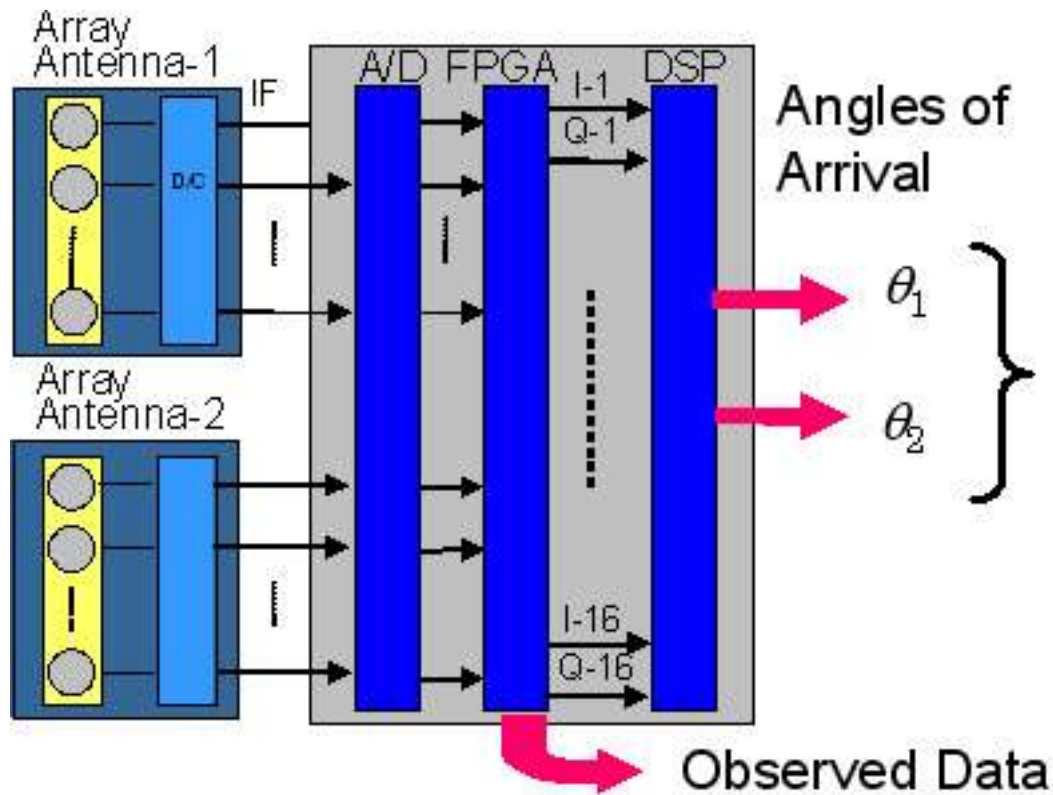
Note: only 8 elements are used for each antenna array because of limitation of data recorder.



Specifications of the transmitting antenna

- Antenna elements : patch element
- Antenna gain : 4 dBi (front)
- Polarization : RHCP

Experiment - 3



- Specifications of ADC, FPGA and DSP

- ADC : 14 bit, 16 ch

Location - FPGA : XC2V3000,
3000000 Gates

- DSP : TI C6701

Block diagram of Rx. experiment system

Array Calibration - 1

(Amplitude and phase compensation technique)

- The calibration matrix can be constructed by differences between the measurement amplitude and phase of each element and the ideal ones.
 - Signal impinging on the antenna array of 0 degrees
 - the signal amplitude and phase of each element are theoretically same,

Amplitude: $\alpha_1 = \alpha_2 = \dots = \alpha_m$ Phase: $\phi_1 = \phi_2 = \dots = \phi_m$

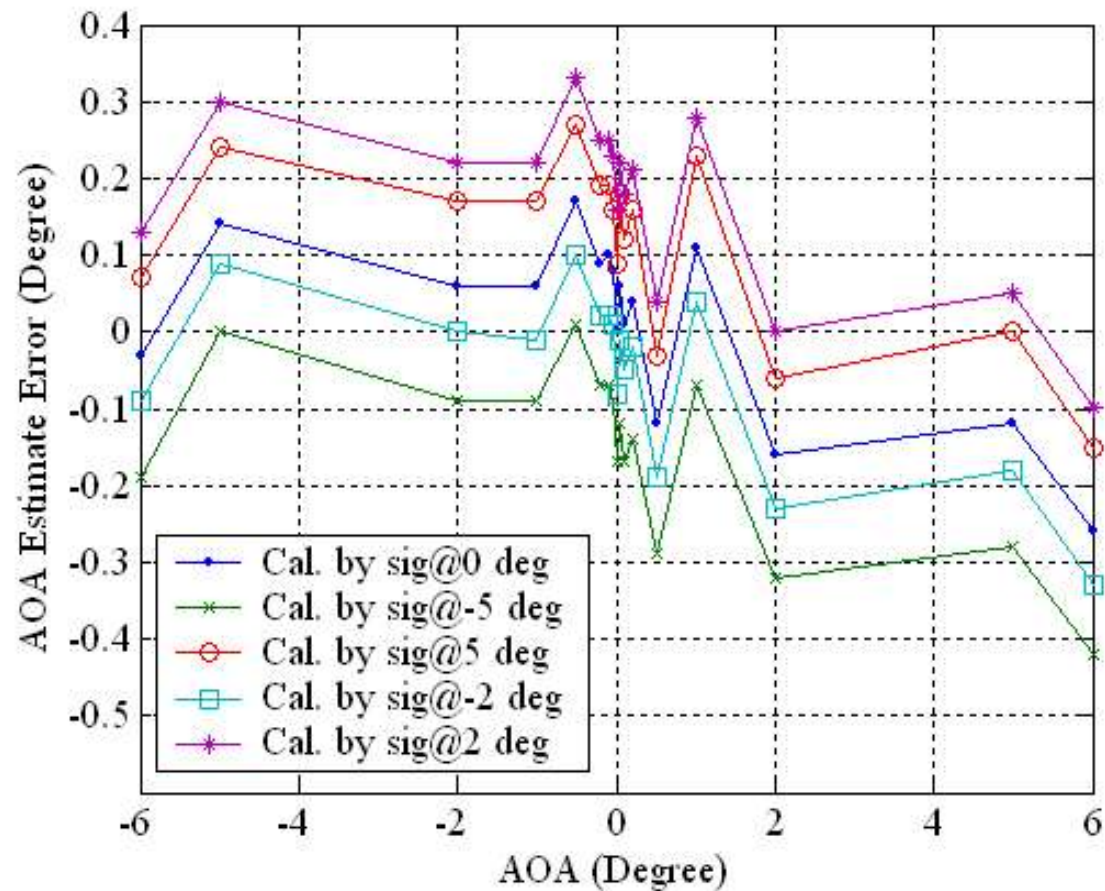
- considered as the ideal amplitude and phase of each element.
- Therefore, the calibration matrix can be obtained by

$$C = \text{diag} \left[1, \frac{\alpha_1}{\alpha_2} e^{j(\phi_1 - \phi_2)}, \dots, \frac{\alpha_1}{\alpha_m} e^{j(\phi_1 - \phi_m)} \right]$$

where the first element is considered as the reference.

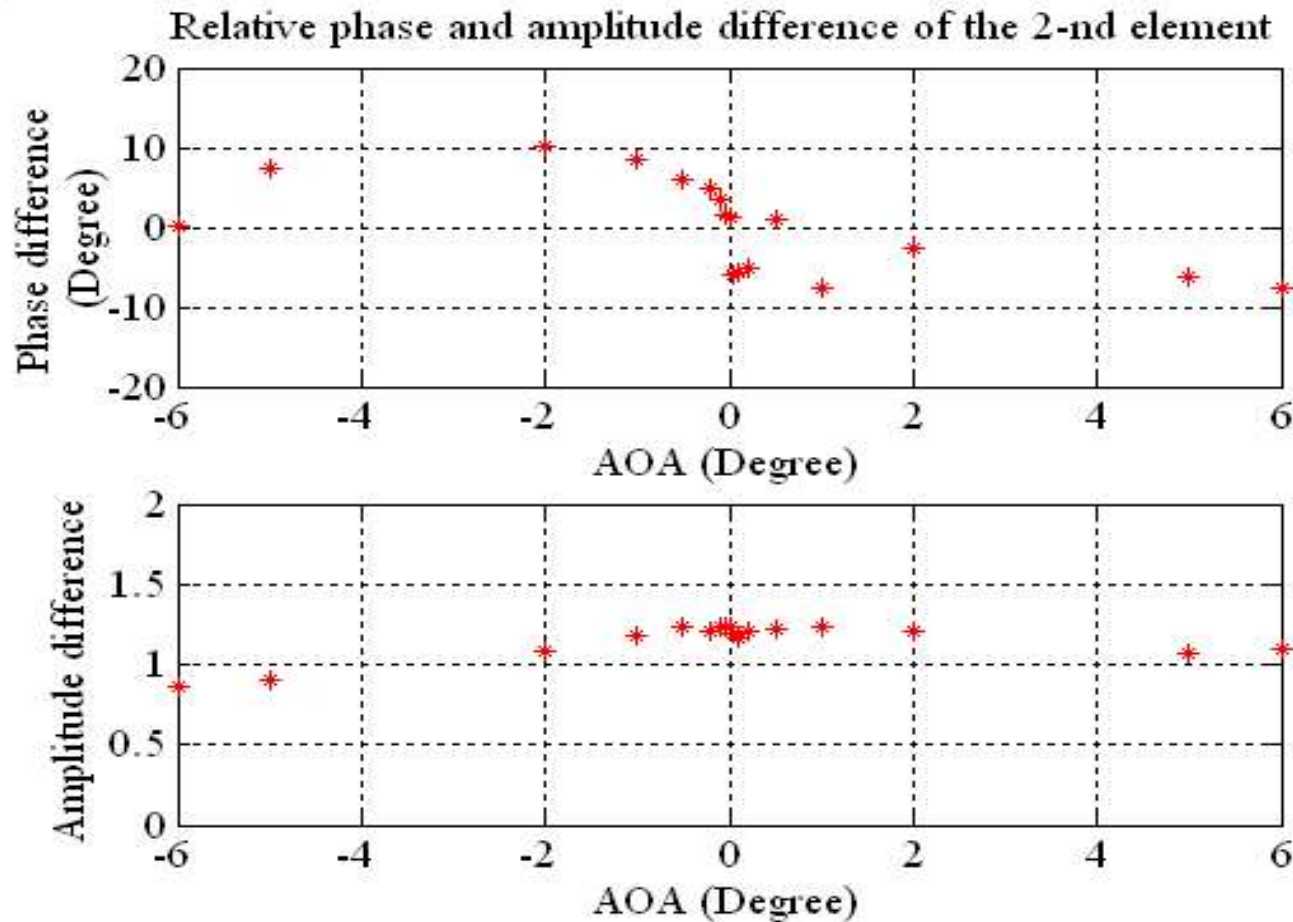
Result of AOA Estimation Error - 1

- Using calibration data computed by the measurement signals of which AOA are -5, -2, 0, 2 and 5 degree, respectively.



Each calibration data is only effective to estimate AOAs near its corresponding degree.

Phase and amplitude difference of the calibration data

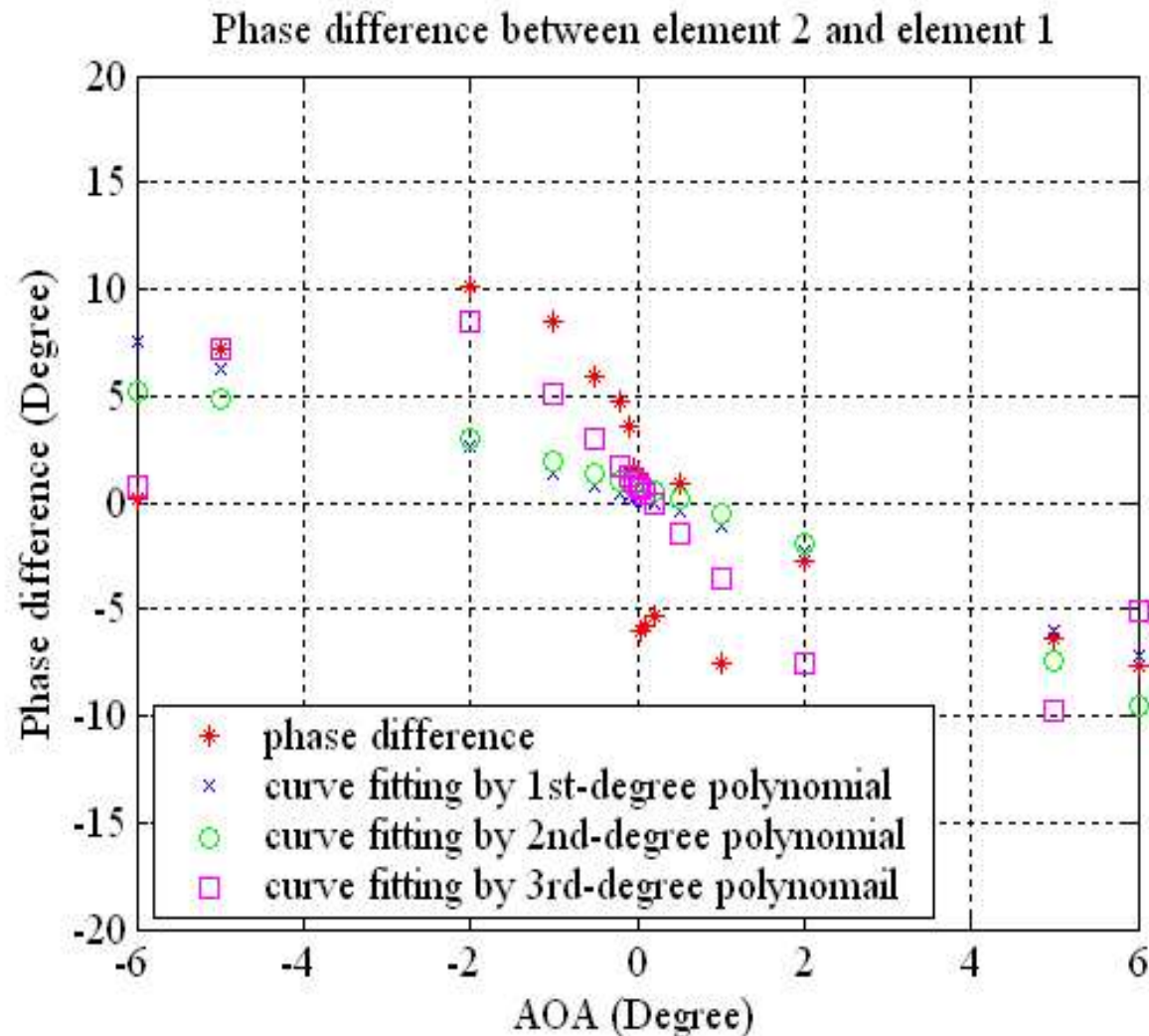


Phase difference of calibration data changes with AOAs, but amplitude difference does not change with AOA.

Array Calibration – 2

(Phase approximation based on least square problem)

LS data fitting by the first-, the second- and the third-degree polynomials



LS data fitting by the first-degree polynomial

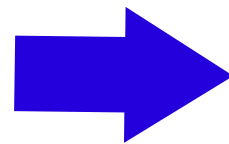
- To modify calibration data by applying the least square approach to approximate phase differences
 - Phase difference of each element is approximated by the first-degree polynomial;

$$\phi_{m,1st}(\theta) = \tilde{a}_{1m}\theta + \tilde{a}_{0m}.$$

$\tilde{a}_{1m}$ and $\tilde{a}_{0m}$ are m -th element unknown parameters approximated by

$$[\tilde{a}_{1m}, \tilde{a}_{0m}]^T = (\Theta_{1st}^T \Theta_{1st})^{-1} \Theta_{1st}^T \Phi_m,$$

$$\Theta_{1st} = \begin{bmatrix} \theta_1 & 1 \\ \theta_2 & 1 \\ \vdots & \vdots \\ \theta_L & 1 \end{bmatrix}, \Phi_m = \begin{bmatrix} \phi_{m,1} \\ \phi_{m,2} \\ \vdots \\ \phi_{m,L} \end{bmatrix}.$$



$$C(\theta) = \text{diag}[C_1(\theta), \dots, C_M(\theta)];$$

$$C_m(\theta) = \alpha_{m0} e^{j\phi_{m,1st}(\theta)}$$

LS data fitting by the second-degree polynomial

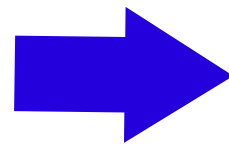
- Phase difference of each element is approximated by the second-degree polynomial;

$$\phi_{m,2nd}(\theta) = \tilde{b}_{2m}\theta^2 + \tilde{b}_{1m}\theta + \tilde{b}_{0m}.$$

$\tilde{b}_{2m}$, $\tilde{b}_{1m}$ and $\tilde{b}_{0m}$ are three unknown parameters of m -th element;

$$[\tilde{b}_{2m}, \tilde{b}_{1m}, \tilde{b}_{0m}]^T = (\Theta_{2nd}^T \Theta_{2nd})^{-1} \Theta_{2nd}^T \Phi_m,$$

$$\Theta_{2nd} = \begin{bmatrix} \theta_1^2 & \theta_1 & 1 \\ \theta_2^2 & \theta_2 & 1 \\ \vdots & \vdots & \vdots \\ \theta_L^2 & \theta_L & 1 \end{bmatrix}, \quad \Phi_m = \begin{bmatrix} \phi_{m,1} \\ \phi_{m,2} \\ \vdots \\ \phi_{m,L} \end{bmatrix}.$$



$$C(\theta) = \text{diag}[C_1(\theta), \dots, C_M(\theta)];$$

$$C_m(\theta) = \alpha_{m0} e^{j\phi_{m,2nd}(\theta)}$$

LS data fitting by the third-degree polynomial

- Phase difference of each element is approximated by the third-degree polynomial;

$$\phi_{m,3rd}(\theta) = \tilde{c}_{3m}\theta^3 + \tilde{c}_{2m}\theta^2 + \tilde{c}_{1m}\theta + \tilde{c}_{0m}.$$

$\tilde{c}_{3m}$, $\tilde{c}_{2m}$, $\tilde{c}_{1m}$ and $\tilde{c}_{0m}$ are three unknown parameters of m -th element;

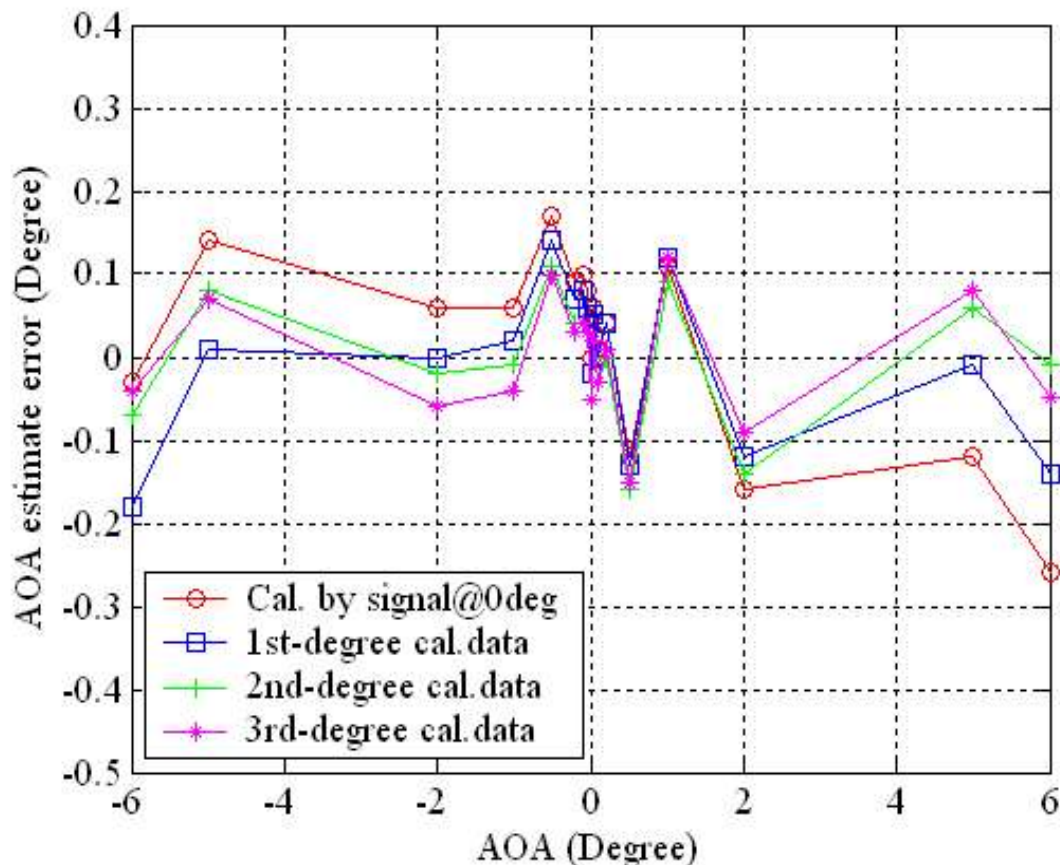
$$[\tilde{c}_{3m}, \tilde{c}_{2m}, \tilde{c}_{1m}, \tilde{c}_{0m}]^T = (\Theta_{3rd}^T \Theta_{3rd})^{-1} \Theta_{3rd}^T \Phi_m,$$

$$\Theta_{3rd} = \begin{bmatrix} \theta_1^3 & \theta_1^2 & \theta_1 & 1 \\ \theta_2^3 & \theta_2^2 & \theta_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \theta_L^3 & \theta_L^2 & \theta_L & 1 \end{bmatrix}, \Phi_m = \begin{bmatrix} \phi_{m,1} \\ \phi_{m,2} \\ \vdots \\ \phi_{m,L} \end{bmatrix} \quad \rightarrow \quad \boxed{C(\theta) = \text{diag}[C_1(\theta), \dots, C_M(\theta)]};$$

$$C_m(\theta) = \alpha_{m0} e^{j\phi_{m,3rd}(\theta)}$$

Result of AOA Estimation Error - 2

- Result of AOA estimation error when calibration matrix computed by the measurement signals of which AOA is 0 degrees and calibration data applying LS, respectively.



The higher the degree LS fitting is used to estimate phase diff., the better improvement is obtained.

The result after applying LS with phase of calibration data is not good as expected (< 0.15 degree).

Conclusion

- The LS approximation of phase differences to calibration data seems to be effective for AOA estimation with the higher degree approximation.
- Estimation errors in some AOAs are still high (expect to less than 0.15 degree).
 - This might be due to imperfect calibration data affected, for instance, by electromagnetics diffraction or instability of the antenna array.
- To correct the calibration error, the properties of the antenna array are needed to be clarified for further works.